Centre of mass calculations

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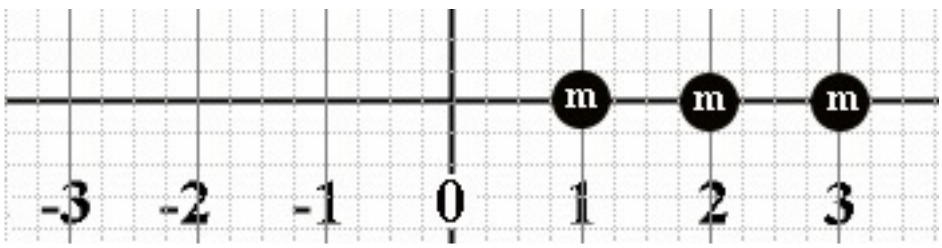
Why does a globular cluster not collapse over time because of gravitational attraction between the stars?

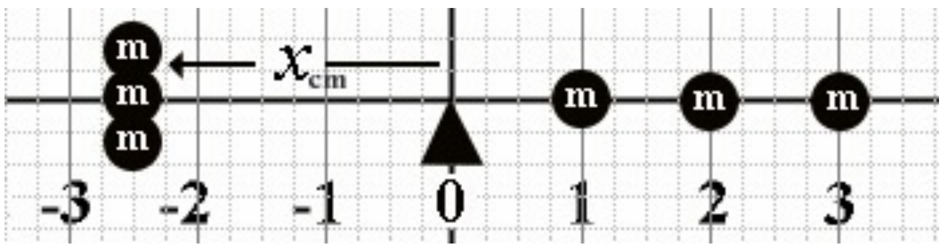
The answer has it that each star is orbiting the centre of mass in its own unique orbit and the spherical cluster is stable in the same way that the solar system is stable, but what exactly is the *centre of mass*?

Example 1

Three equal masses are on a number line.



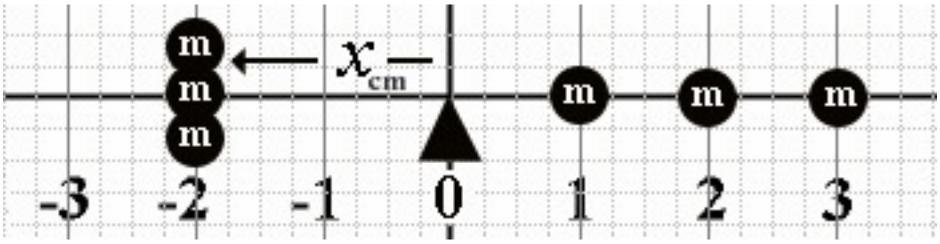
To find x_{cm}, the distance to the centre of mass from (0, 0), we multiply each mass by the distance from (0, 0) and take the sum to balance the bar.



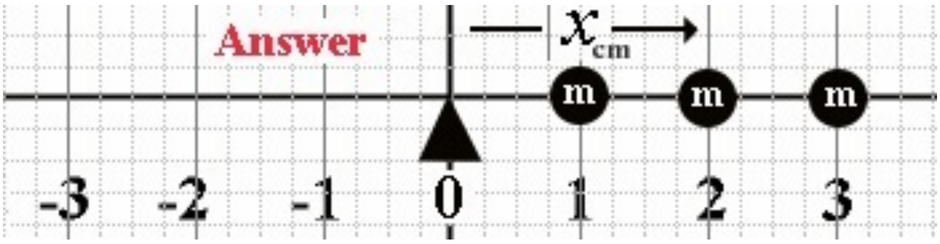
$$x_{cm} \times 3m = 1m + 2m + 3m$$

$$x_{cm} = m(1 + 2 + 3)/3m$$

$$= 2$$



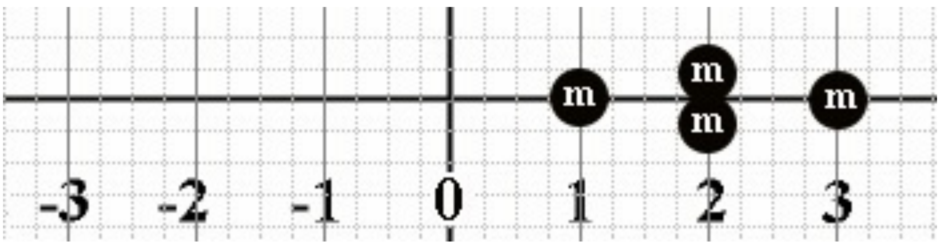
The beam is balanced with three masses at (-2, 0).



The centre of mass is at (2, 0)

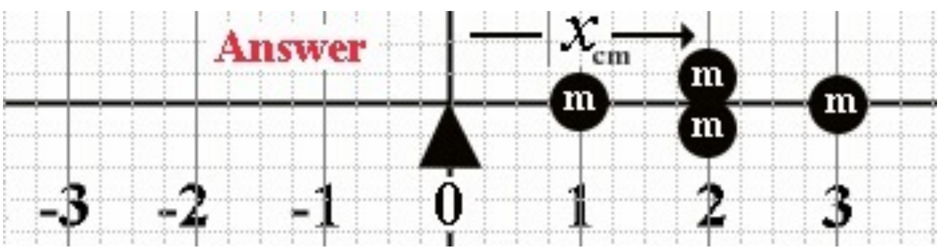
Example 2

Four equal masses are on a number line.



From (0, 0) … $x_{cm} = m(1 + 2 + 2 + 3)/4m$

$= 2$



The centre of mass is at (2, 0)

The general case

The formula for the distance to the centre of mass from any point on the line, x_{cm}, can be written for any number of masses of any size as …

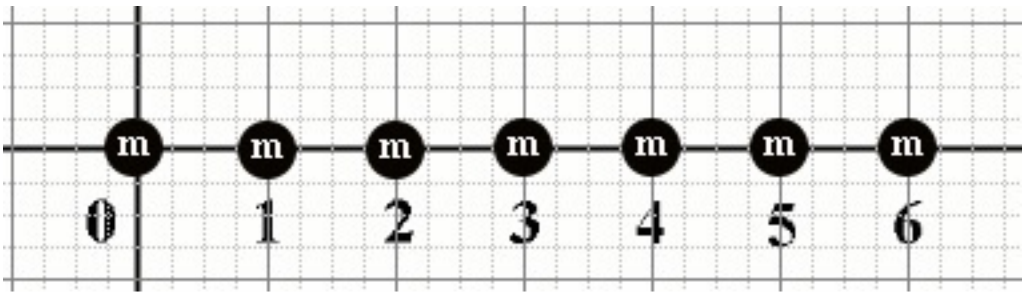
$$x_{cm} = (m_1x_1 + m_2x_2 + \ldots.)/(m_1 + m_2 + \ldots.)$$

If the masses are all the same, the common factor m can be taken out of the top and bottom lines and cancelled and the formula simplifies to …

$$x_{cm} = (x_1 + x_2 + \ldots)/N \quad \text{… where } N \text{ is the number of masses.}$$

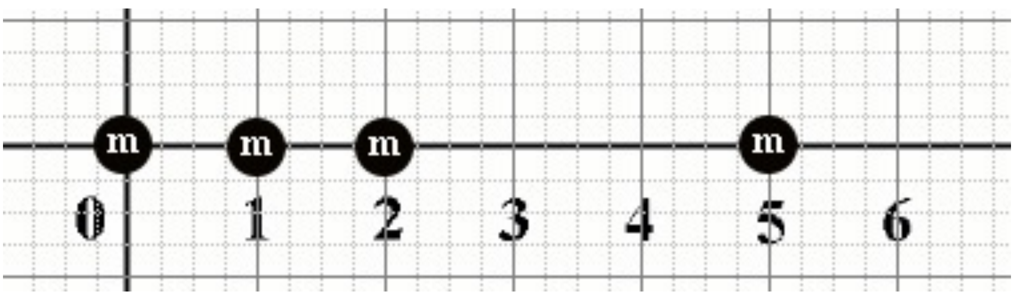
The result is the average of the distances $x_1, x_2, \ldots x_n$.

Examples 3 and 4



The distance from (0, 0) to the centre of mass is the average …

$$(0 + 1 + 2 + 3 + 4 + 5 + 6)/7 = 3$$

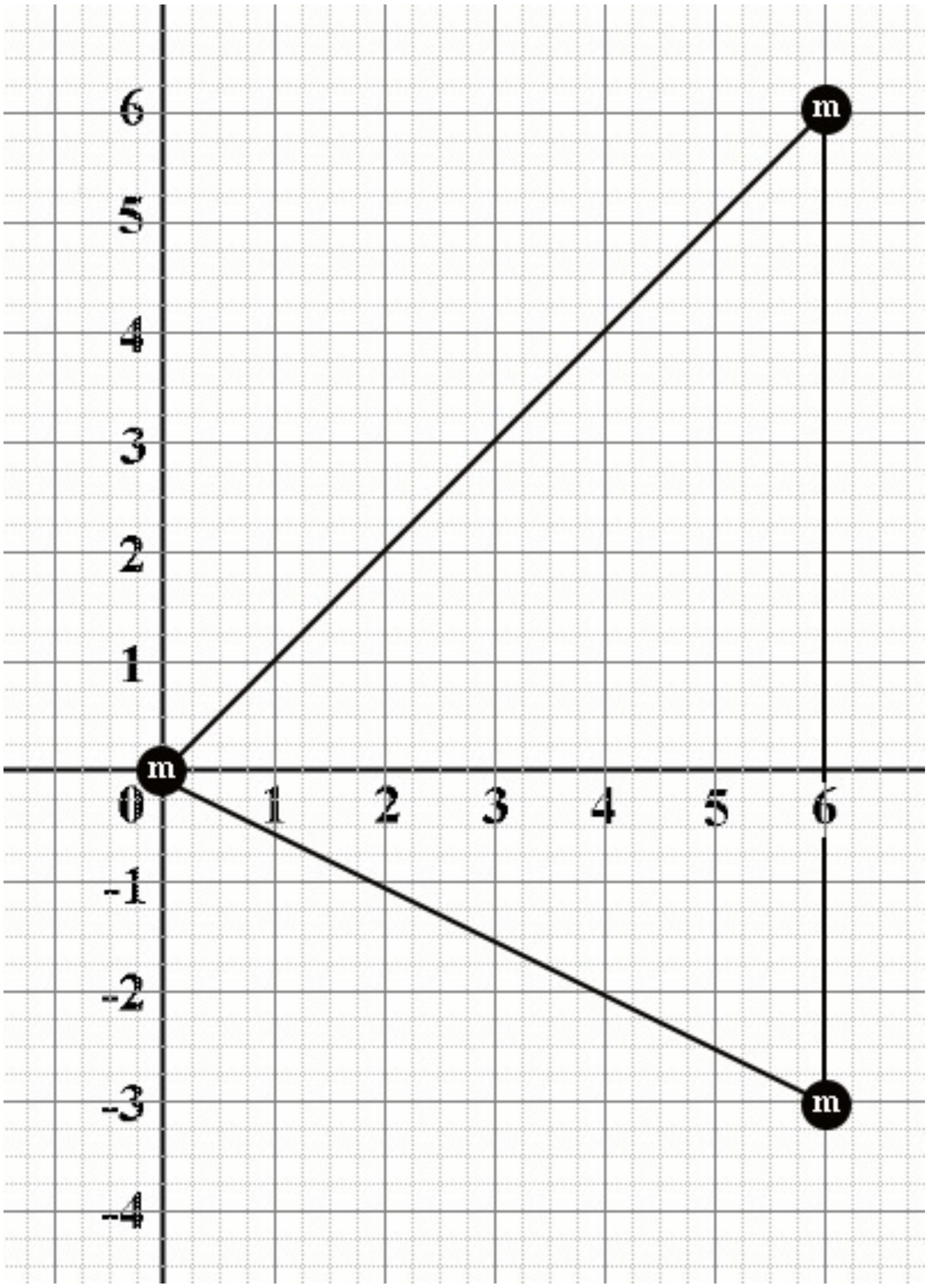


The distance from (0, 0) to the centre of mass is the average …

$$(0 + 1 + 2 + 5)/4 = 2$$

Example 5

Three equal masses are placed on the Cartesian plane

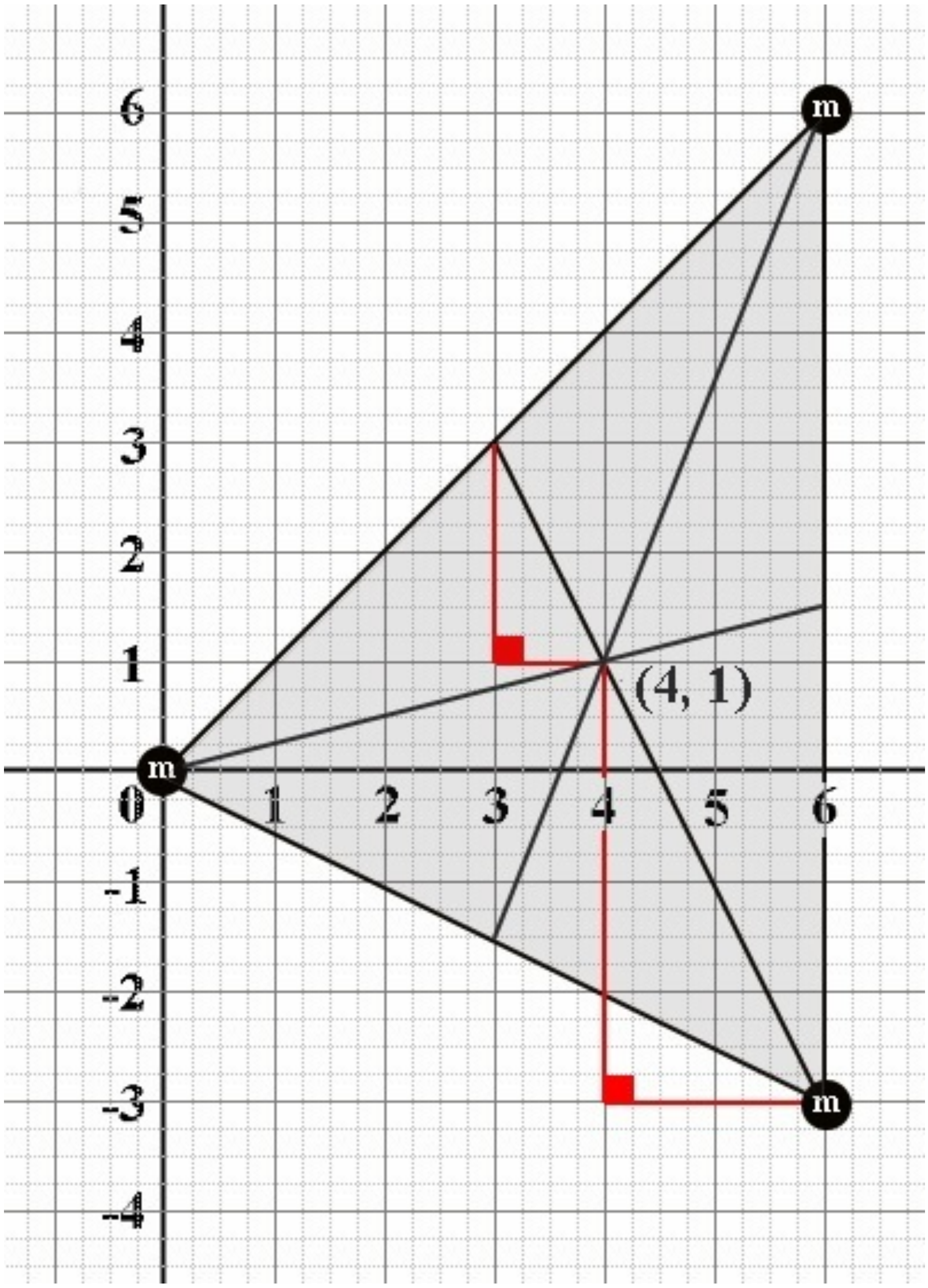


From (0, 0) … $x_{cm} = (0 + 6 + 6)/3 = 4$

From (0, 0) … $y_{cm} = (0 - 3 + 6)/3 = 1$

The centre of mass is at the point (4, 1).

The point (4, 1) is the centroid of the triangle.



The reader may note that the lines intersecting at (4, 1) are medians that intersect the mid point of each side. Right angled triangles have been added in red to show the division of one median in the ratio 2:1. The reader may confirm the remaining intersection ratios by adding their own triangles.

The example shows that the centre of mass of the uniform grey triangular sheet is at the centroid. The mass of the triangle ($3m$) may be replaced by a mass of $3m$ at the centroid … or … by three masses of m at each vertex.