Reflections and Rotations on the Cartesian plane

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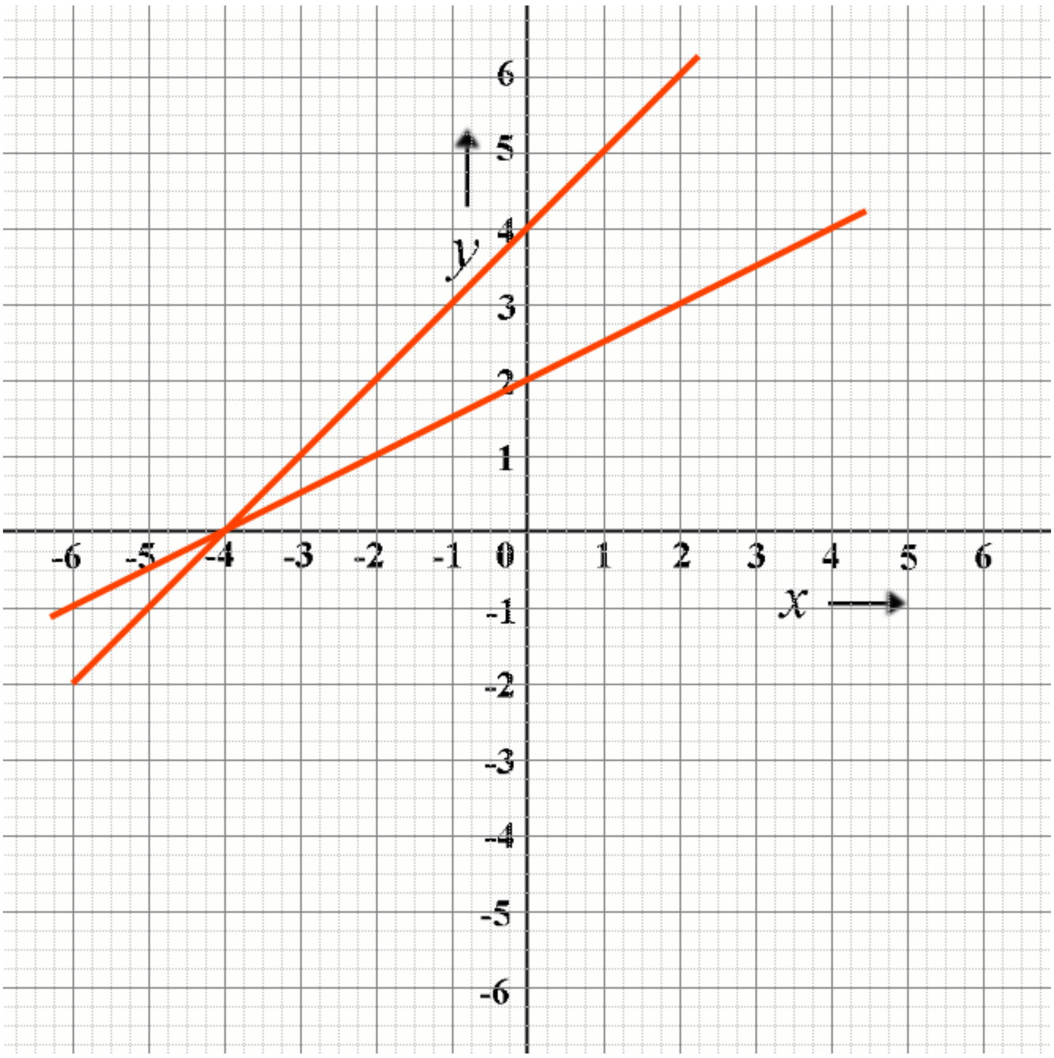
If you read *Transformations* [1] in the mathematics index you will see how to modify equations in x and y to translate and scale figures on the Cartesian plane. In this article we see how to reflect lines and circles in selected mirror lines, and how to rotate figures made with straight lines in steps of $\pm 90^\circ$.

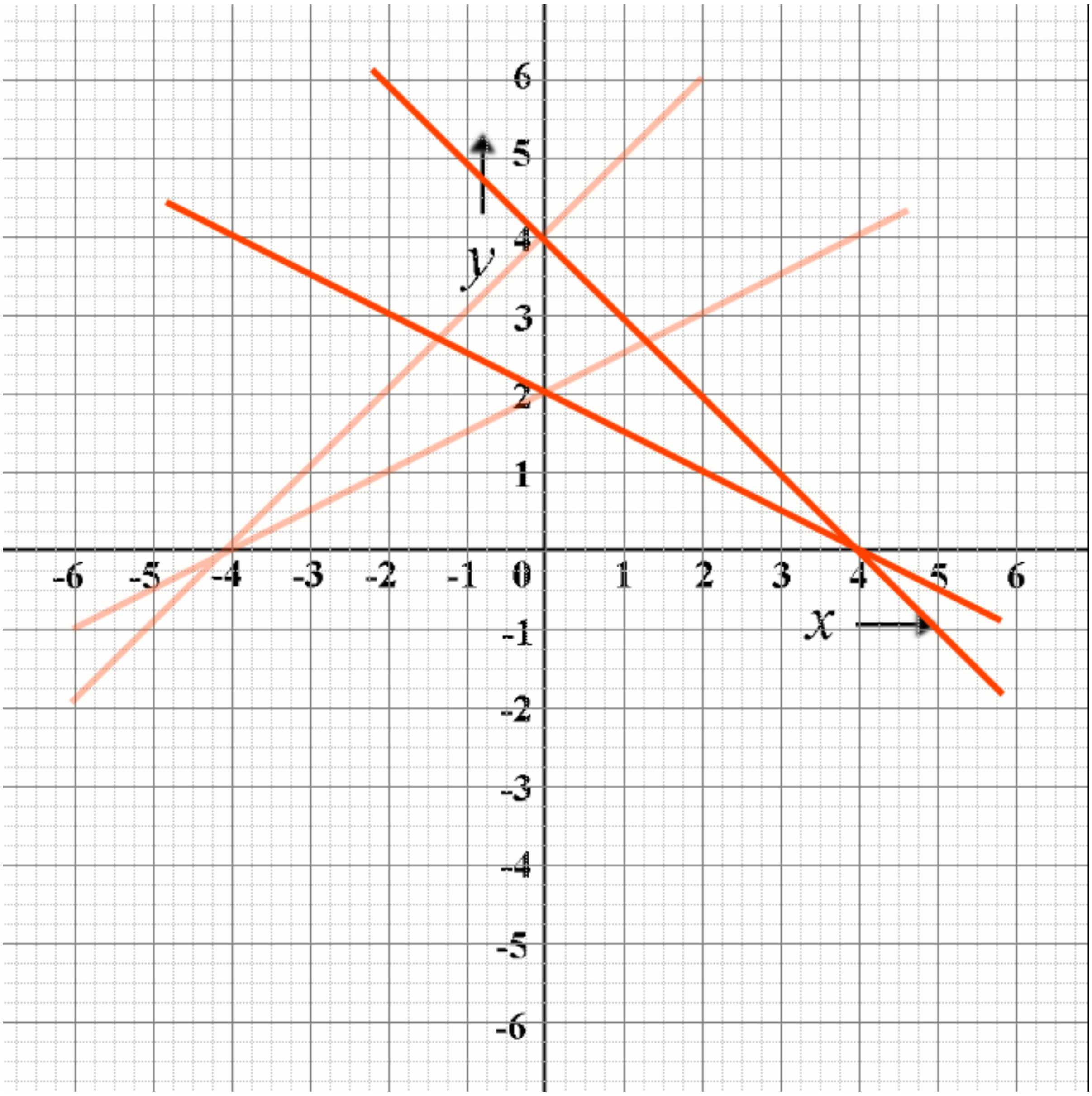
Two lines are drawn on the plane.



Their equations are … $y = x + 4$ …. [1]

… and … $y = {}^1/_2 x + 2$ …. [2]

Reflection in the *y* axis



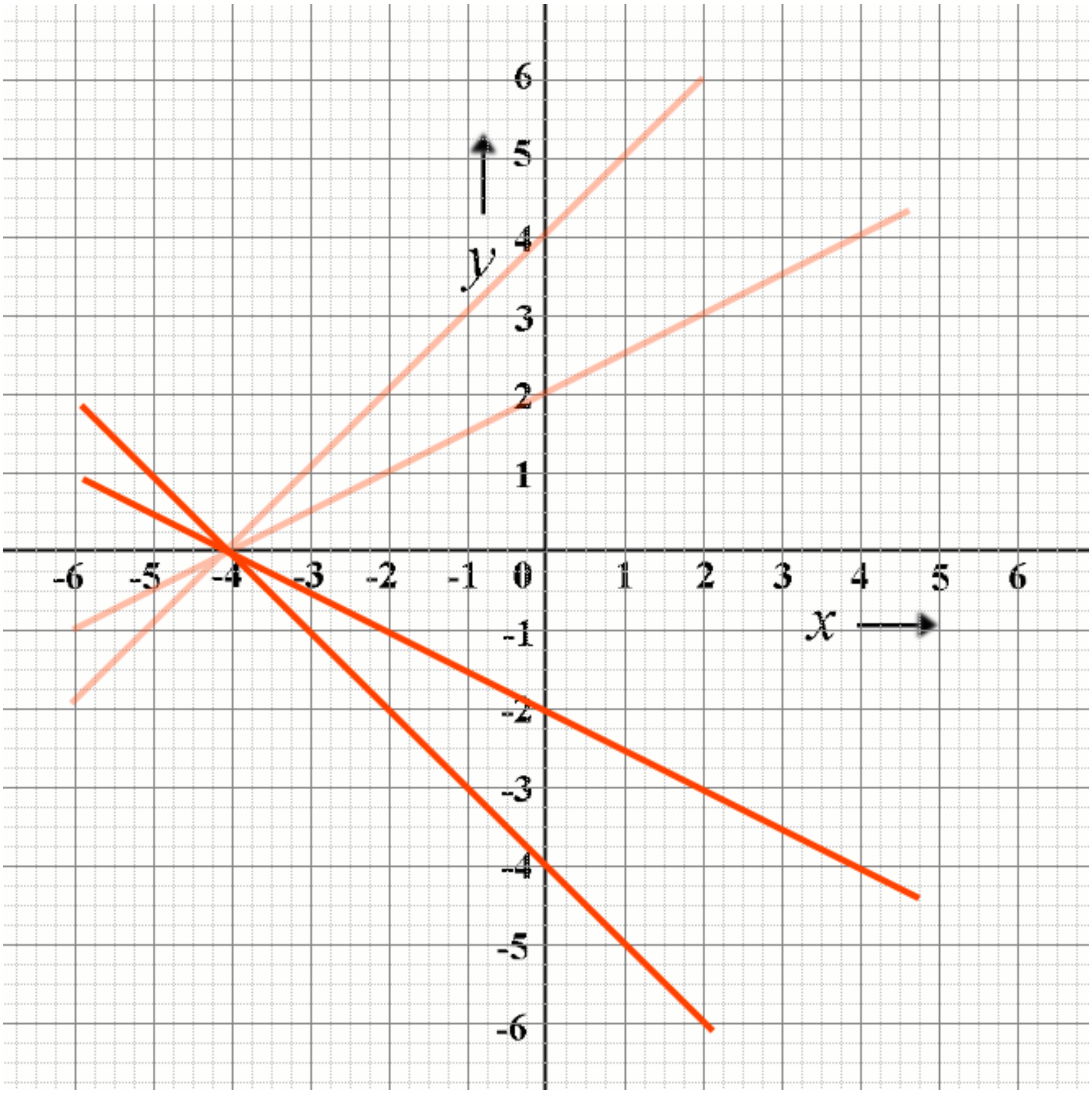
The original lines (in pale red) are reflected in the *y* axis ($x = 0$) by replacing x with $-x$ in equations 1 and 2.

$$y = -x + 4$$

$$y = -{}^{1}/_{2}\,x + 2$$

Note that the x values of all points on any line are converted to $-x$, except for points on the line $x = 0$, which is the mirror line.

Reflection in the *x* axis



The original lines in pale red are reflected in the *x* axis ($y = 0$) by replacing *y* with -*y* in equations 1 and 2.

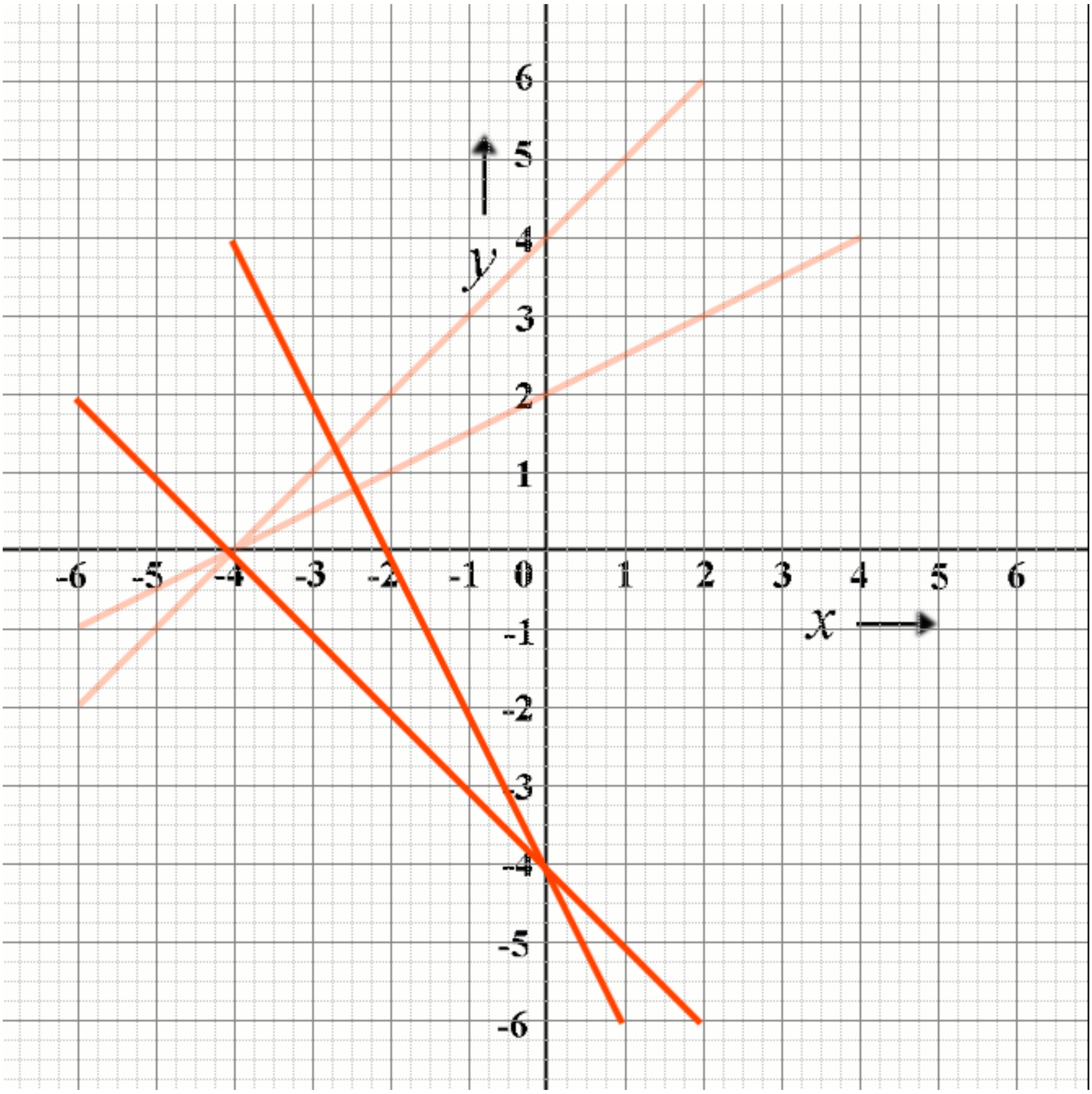
$$-y = x + 4$$

Modified equation … $y = -x - 3$

$$-y = {}^1/_2\,x + 2$$

Modified equation … $y = -{}^1/_2\,x - 2$

Positive rotation about (0, 0) by 90º



The original lines in pale red are rotated by +90º (or -270º) by exchanging $x \Leftrightarrow y$ and replacing x with $-x$ in equations 1 and 2.

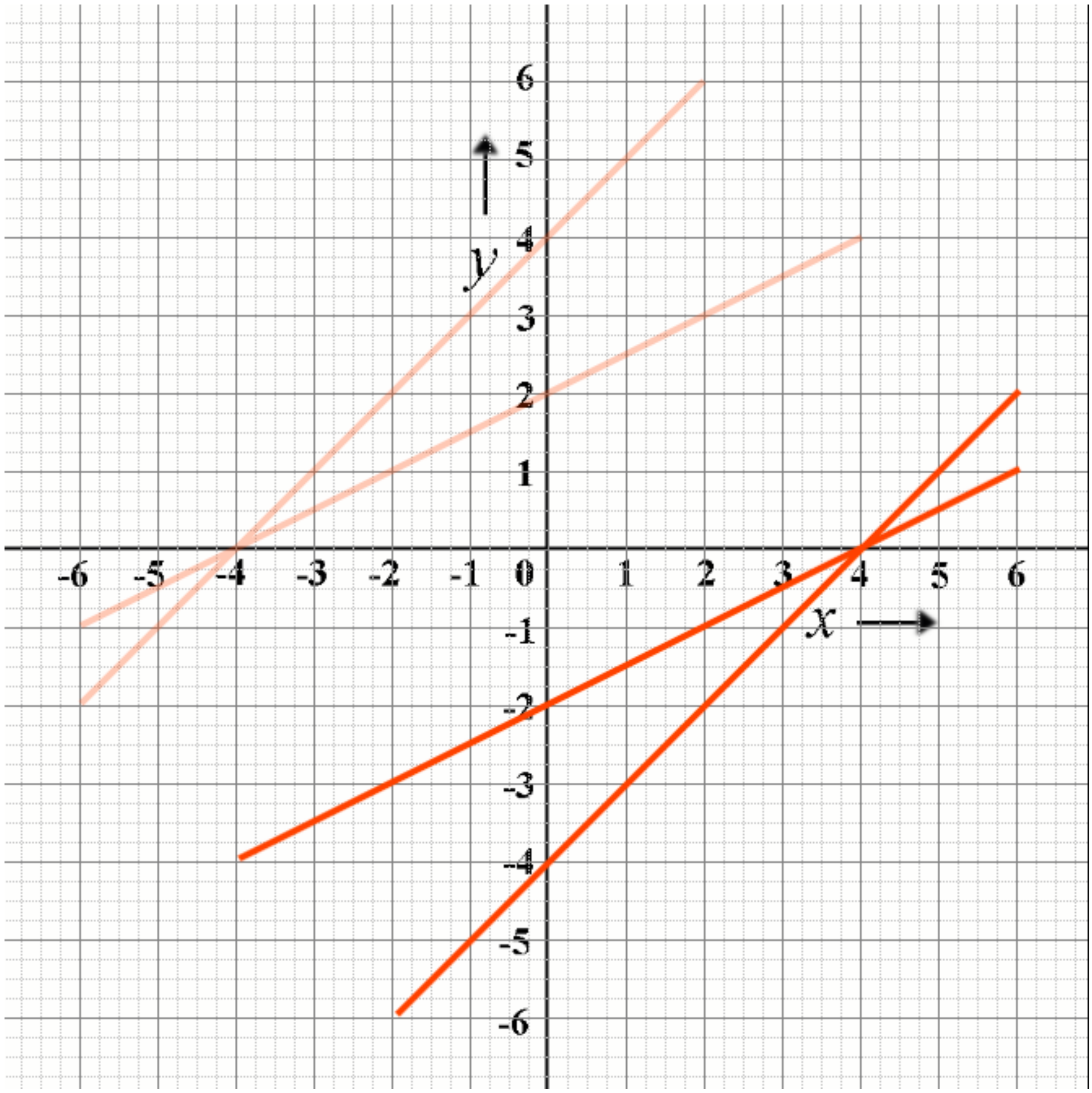
$$-x = y + 4$$

Modified equation … $y = -x - 4$

$$-x = {}^{1}/_{2}\, y + 2$$

Modified equation … $y = -2x - 4$

Positive rotation about (0, 0) by 180º



The original lines (in pale red) are rotated 180º by replacing ***x*** with **-*x*** and ***y*** with **-*y*** in equations 1 and 2.

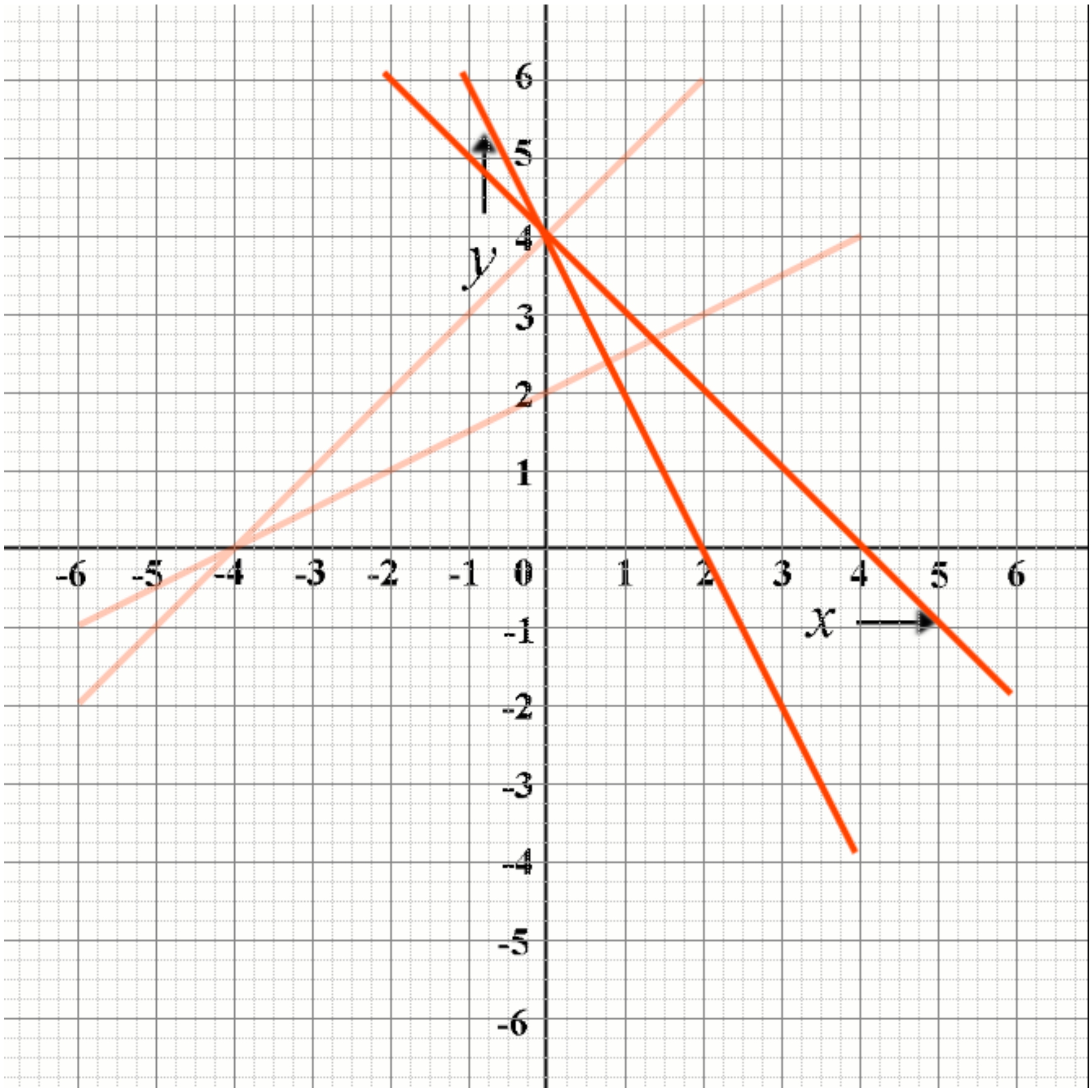
$$-y = -x + 4$$

Modified equation … $y = x - 4$

$$-y = -\,{}^{1}/_{2}\, x + 2$$

Modified equation … $y = {}^{1}/_{2}\, x - 2$

Positive rotation about (0, 0) by 270º



The original lines in pale red are rotated by +270º (or -90º) by exchanging x <=> y and replacing y with -y in equations 1 and 2.

$$x = -y + 4$$

Modified equation … $y = -x + 4$

$$x = -{}^{1}/_{2}\, y + 2$$

Modified equation … $y = -2x + 4$

The following transformations have been used.

1 The transformation $x \rightarrow -x$ reflects lines in the y axis, which is the line $x = 0$ which is left undisturbed.

2 The transformation $y \rightarrow -y$ reflects lines in the x axis, which is the line $y = 0$ which is left undisturbed.

3 Both transformations: $x \rightarrow -x$ and $y \rightarrow -y$ rotates lines 180º about the origin (0, 0), which is the only point on the plane left undisturbed. A rotation of 180º is equivalent to refection in the y axis followed by reflection in the x axis.

4 Exchanging $x \Leftarrow\Rightarrow y$ and transforming $x \rightarrow -x$ rotates lines about the origin by +90º (or -270º).

5 Exchanging $x \Leftarrow\Rightarrow y$ and transforming $y \rightarrow -y$ rotates lines about the origin by -270º (or +90º).

Note : the first three transformations are easily understood but the details of 4 and 5 may be difficult to remember. They can be recalled with a simple example, for instance by exchanging $x \Leftarrow\Rightarrow y$ and rotating the line $y = x + 1$, if they are needed and have been forgotten.

Two possible transformations remain.

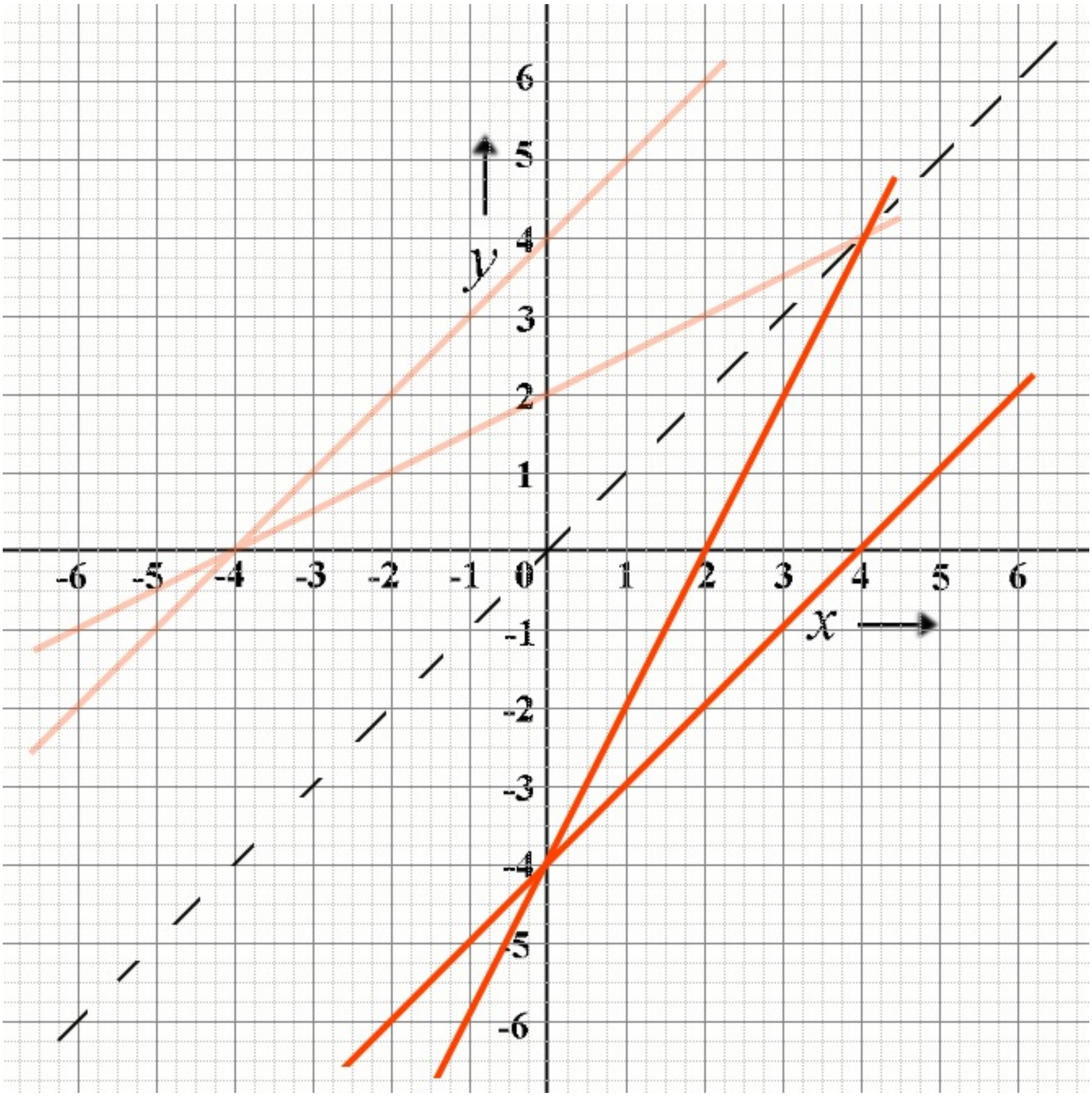
6 Exchanging $x \Leftarrow\Rightarrow y$ ... leaving both signs unchanged.

7 Exchanging $x \Leftarrow\Rightarrow y$ …and transforming $x \rightarrow -x$ and $y \rightarrow -y$.

Along with leaving x and y unchanged, which is a rotation of 360º about the origin, options 6 and 7 complete the eight possible permutations of x and y with positive or negative signs.

The effects of 6 and 7 are shown below.

Reflection in the line $y = x$



The original lines (in pale red) are reflected in the line $y = x$ by replacing y with x and x with y in equations 1 and 2.

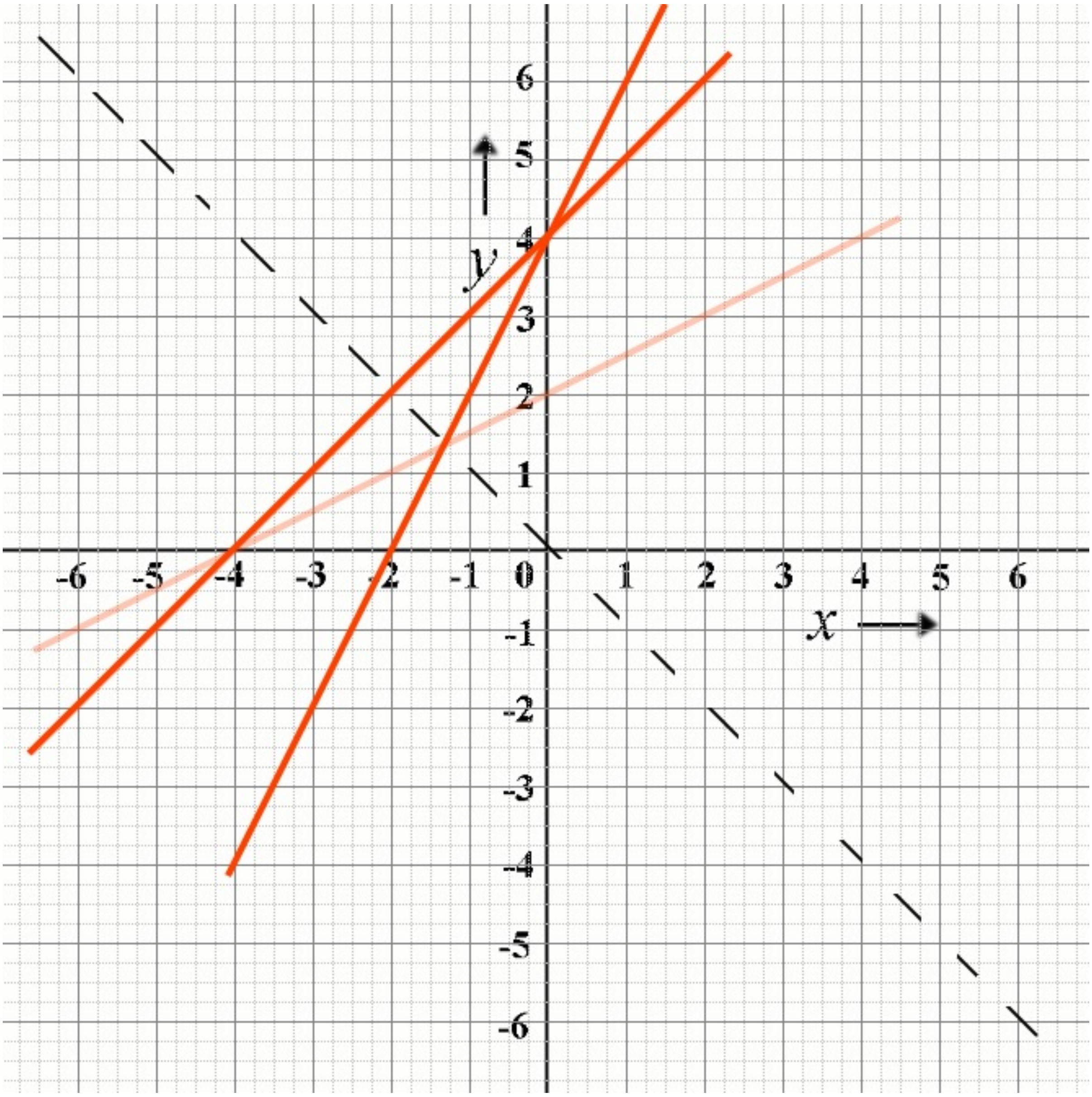
$$x = y + 4$$

Modified equation … $y = x - 4$

$$x = {}^{1}/_{2}\, y + 2$$

Modified equation … $y = 2x - 4$

Reflection in the line $y = -x$



The original lines (in pale red) are reflected in the line $y = -x$ by replacing y with $-x$ and x with $-y$ in equations 1 and 2.

$$-x = -y + 4$$

Modified equation … $y = x + 4$

$$-x = -\tfrac{1}{2}y + 2$$

Modified equation … $y = 2x + 4$