Ray diagrams

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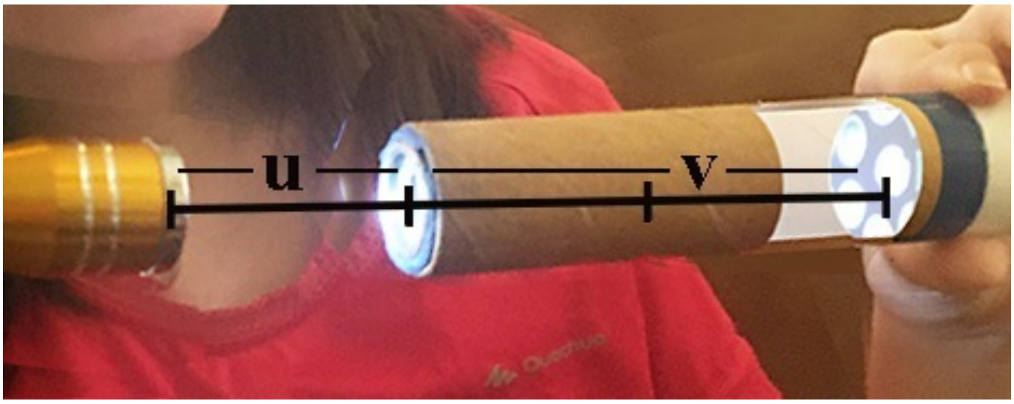
The article *A microscope* was written in response to a teacher's question in lockdown. *Explain how a compound microscope works*. That response did not provoke a second question so it satisfied his request, but the explanation was partial. I did not explain how it is that a convex lens can be used as both a projector to form an enlarged real image *and* as a magnifying glass, or this more difficult question: how exactly does a magnifying glass do what it does?

Proper explanations require ray diagrams.

Definitions



We begin with the enlarged image inside the barrel of the microscope.

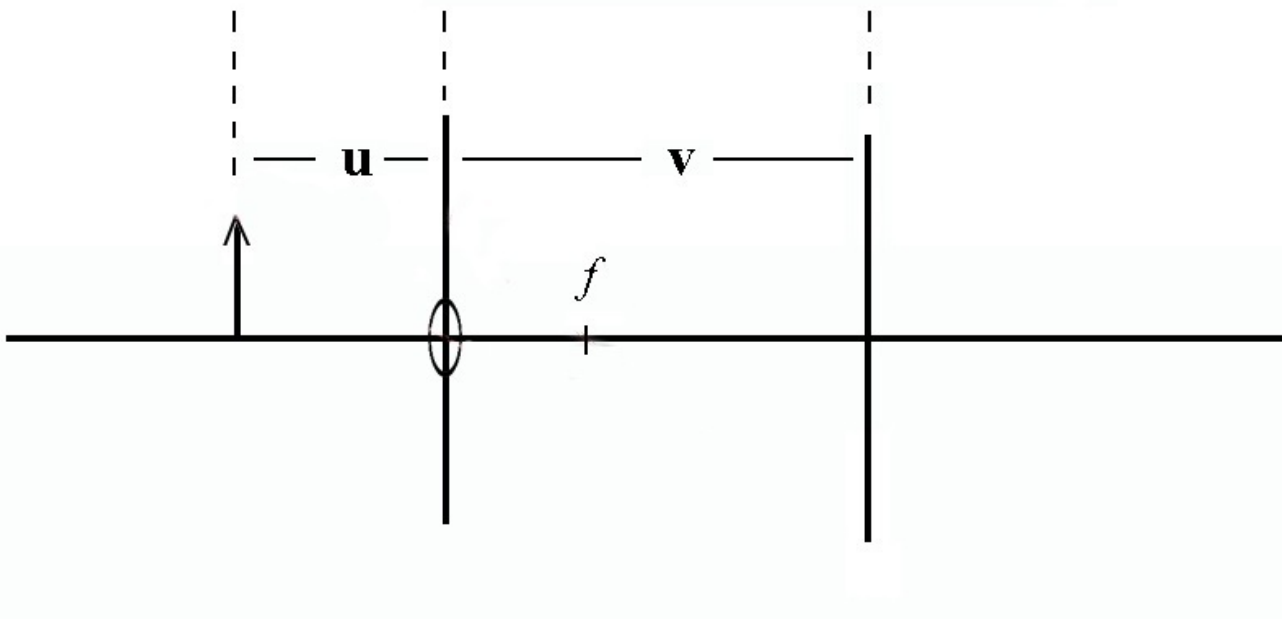
The *object distance*, from the *objective lens* at the left hand end of the barrel above to the front plate of the torch, is labelled u. The *image distance*, from the *objective lens* to the bright enlarged image on the paper screen inside the barrel, is labeled v. Some books use other symbols but we have chosen to use the traditional variables u and v.

Measuring on the photograph shows that in this case $v = 2u$. That is not accidental. It has been arranged this way to simplify what follows.

The objective lens

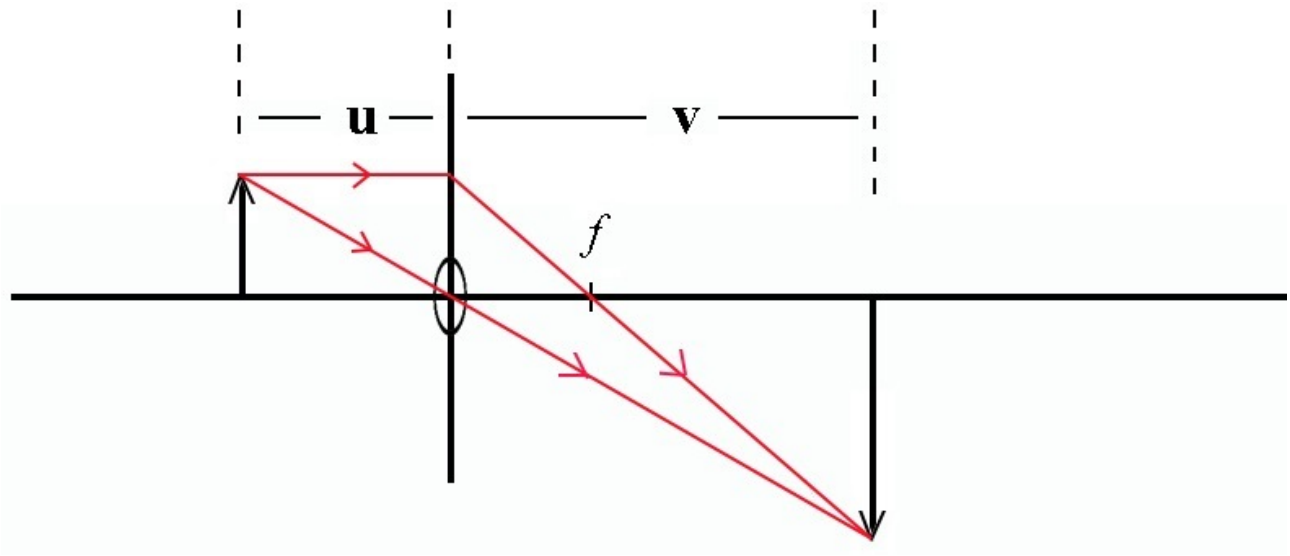
A photograph has been clear-cut to show the positions of the object, the objective lens, and the bright image in the barrel.

To construct the ray diagram we first draw a horizontal axis, an arrow to represent the object, a line to represent the lens with a symbol to show its shape, the focal length of the lens and known image position.



The known size of the object is shown by the length of the arrow on the left. The focal length of the lens (found most easily by focusing the image of a distant object on paper) is marked on the axis. The image position is shown as a vertical line. The object distance (symbol u) and image distance (symbol v) are defined on the diagram.

Rays from the tip of an object converge to the tip of the image.

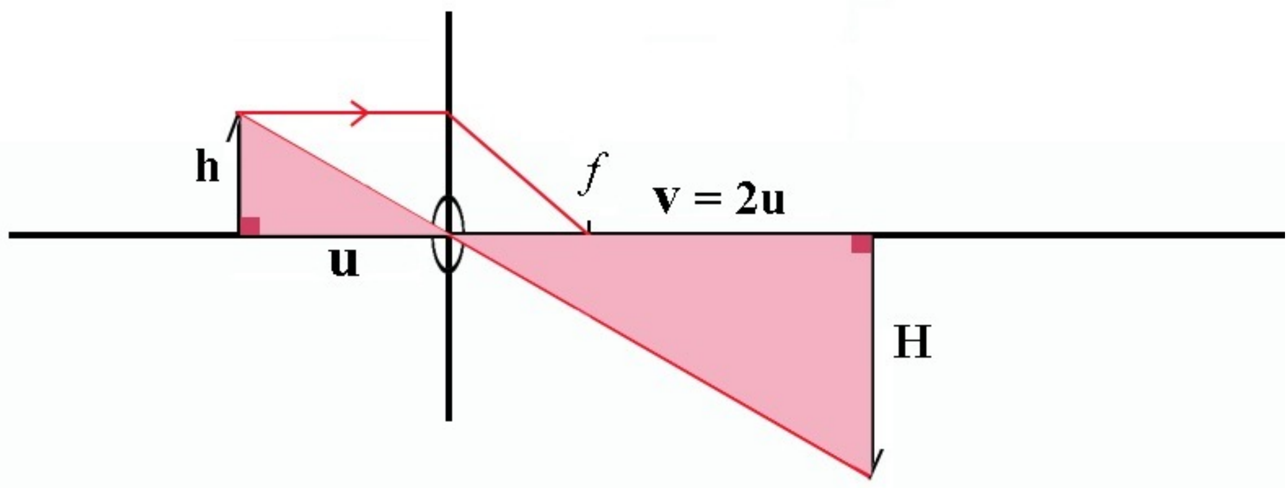


1 The ray to the centre of the lens passes almost straight through.

2 The ray parallel to the axis is bent downward at each surface of the lens and passes through the focal point.

The rays meet at what must be the tip of the image.

The pink triangles are *similar,* they have the same angles.

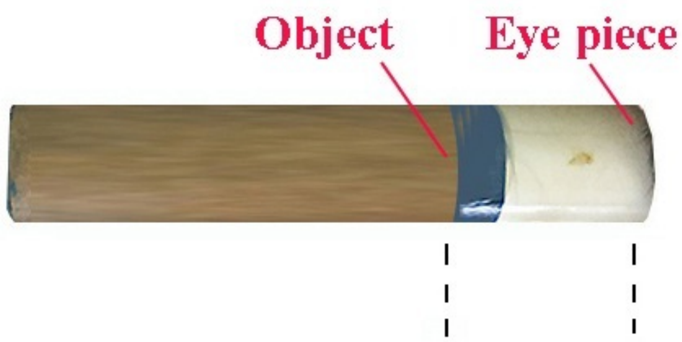


The ratio h/u must equal the ratio $H/2u$.

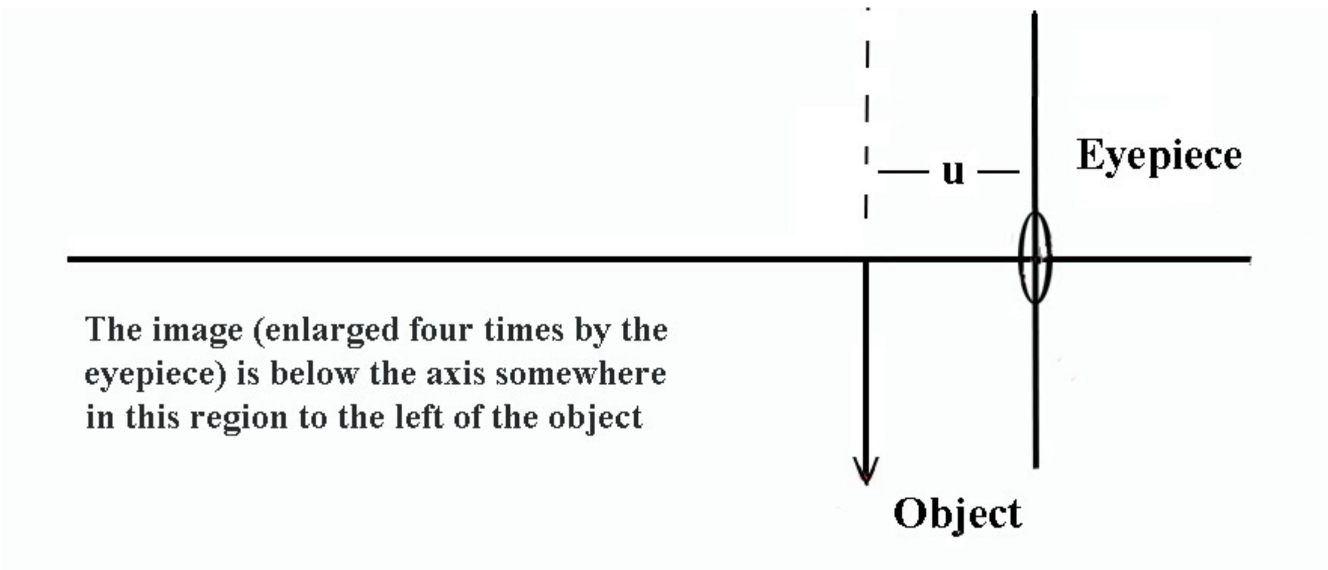
The magnification (defined as H/h) is x2.

The eyepiece lens

The eyepiece is a convex lens used as a magnifying glass. The image shows a loupe (a magnifying glass without a handle) at the right hand end of the barrel.

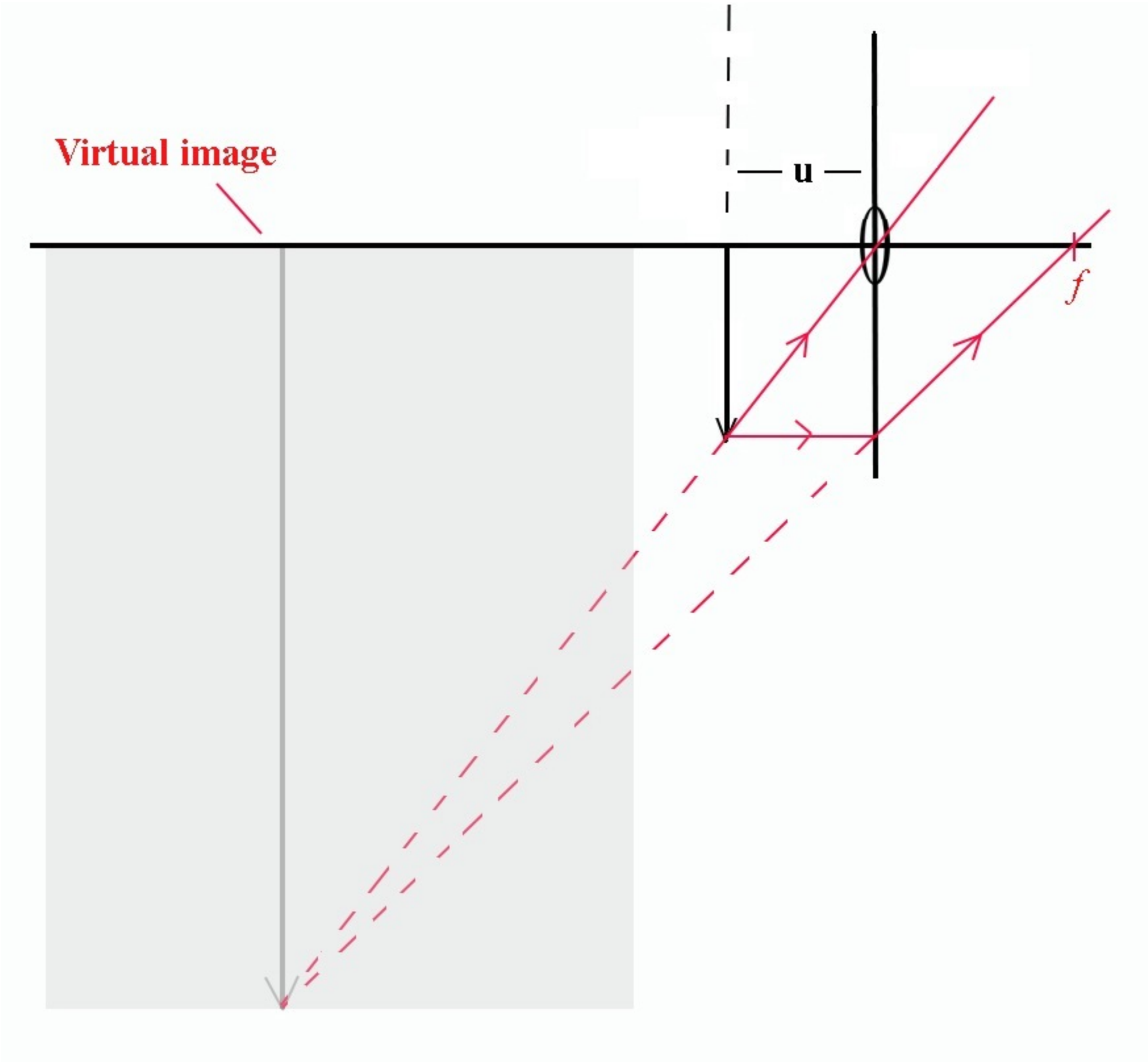


The object for the magnifying glass is the inverted enlarged image formed in the barrel of the microscope by the objective lens. To construct the ray diagram for the eyepiece we first draw what we know to scale.



We know the size of the final image but not the image distance or the focal length of the lens.

We construct the ray diagram as follows …

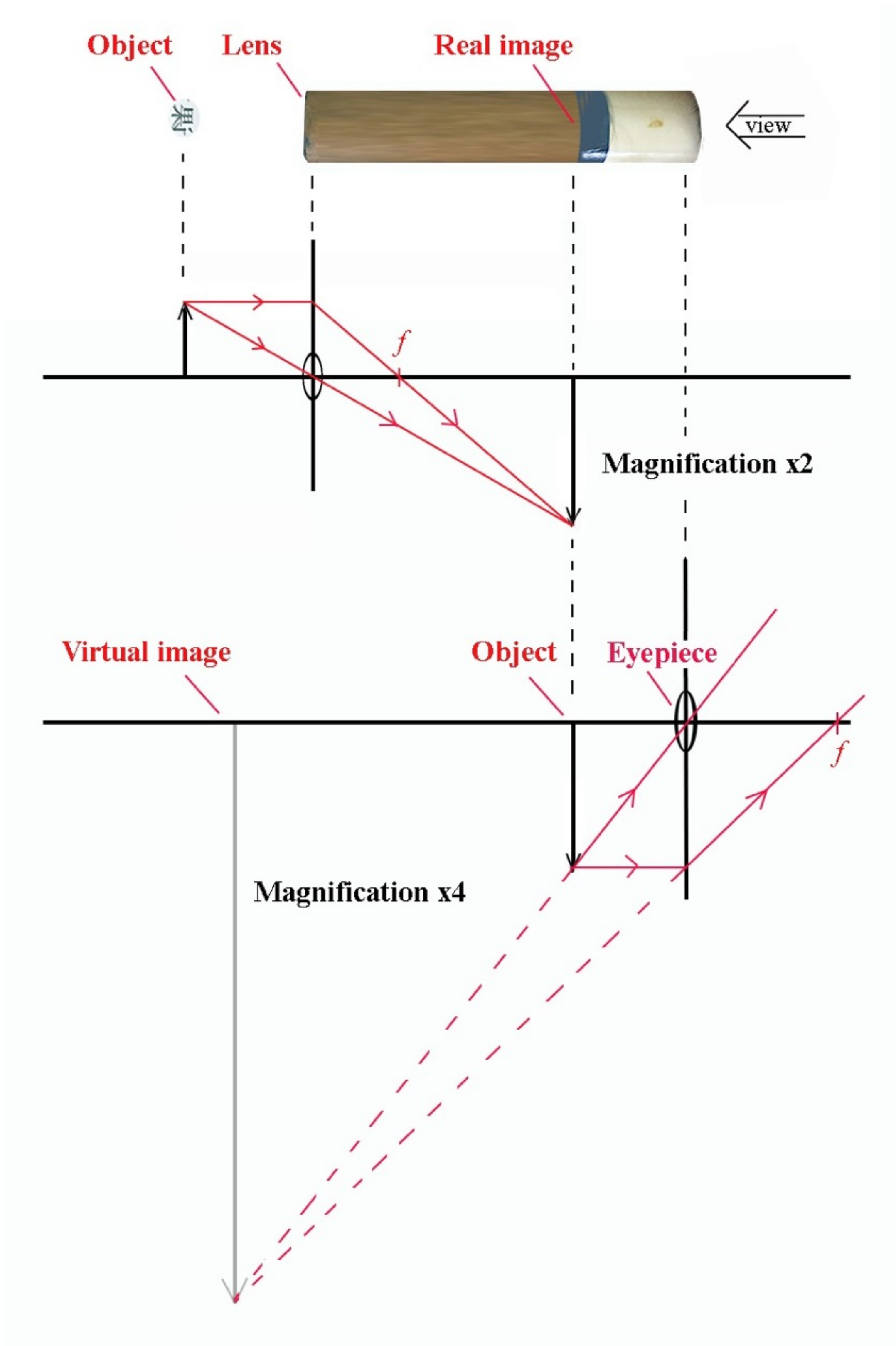


1 Because the magnification is known to be x4, the image is located by drawing the ray from the tip of the object through the centre of the lens. This ray must appear to have come from the tip of the image, that must fit somewhere in the grey rectangle.

2 Completing the ray parallel to the axis that appears to come from the tip of the image (which we have now located) gives the focal length of the eyepiece, where it crosses the axis after passing through the eyepiece lens.

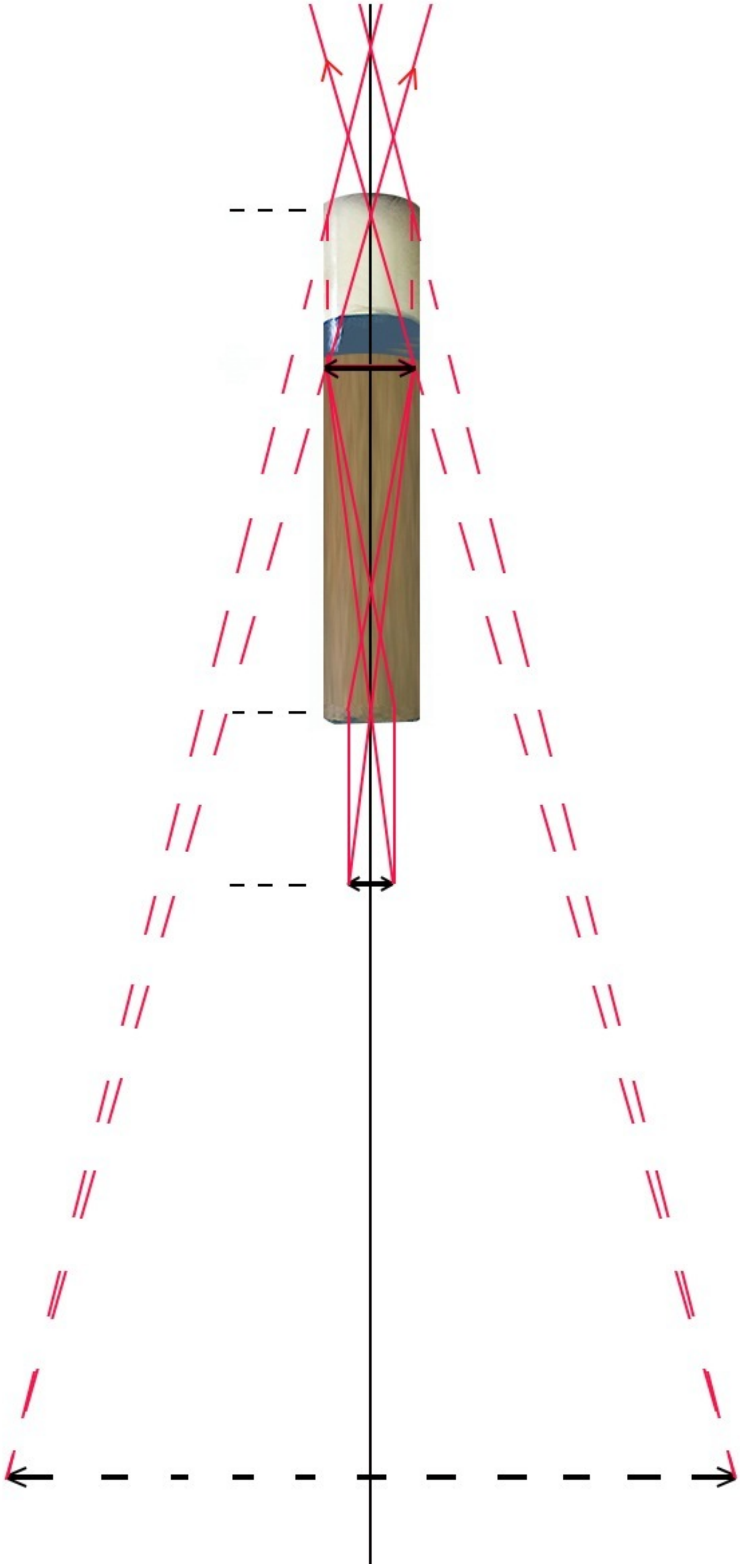
The image in the eyepiece lens is said to be *virtual*. Diverging rays that have passed through the eyepiece only *appear* to have come from this virtual image but have not actually come from it.

All three images to scale



The figure shows the diagrams to scale. Note that the object is close to, but *outside* the focal length of the objective, and *inside* the focal length of the eyepiece. Total magnification is 2x4 = 8.

A conventional ray diagram



Questions

1 On a copy of the ray diagram of a compound microscope:

- label two convex lenses: objective and eyepiece.
- label a real image.
- label a virtual image.
- label object and image distance, *u* ind *v,* for both lenses.
- write down the magnification (M_o) of the objective.
- write down the total magnification of the microscope (M) if the magnification of the eyepiece is x8.
- Label the focal points of the objective and eyepiece, f_o and f_e.

2 *Why* must a real object be inside the focal length of the eyepiece?

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3 Which item in the ray diagram is both an image and an object?

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4 Why is the angular field of view of the microscope larger than the small field of view through the empty barrel?

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Appendix: words to the wise

Magnification

The magnification of the microscope given above as 2x4 = 8 is the *linear* magnification of the final virtual image. By linear magnification we mean the ratio H/h. Magnification is defined in this way so that ray diagrams can be introduced and understood. The magnification that's important in practice when using a microscope is *angular* magnification. By this we mean the ratio of the the angle subtended by the image of the object through the microscope over the angle subtended when viewed at the near point of the naked eye. Angular magnification is more difficult to define and involves the near point distance of the eye.

Typical young people of 10 or 11 have a near point (the closest object distance to the eye for sharp focus) of about 5 cm. Typical parents in their early forties have a near point of about 25 cm. When children read extra fine print they bring the page closer to increase the angle subtended by the letters. Their parents must hold the page at 25 cm or beyond when reading and they must adjust a microscope to move the final image to at least 25 cm so they can view it without blurring. Angular magnification will be discussed in a future article.

Image distortion

A loupe is said to be a "magnifying glass without a handle". That definition serves our purpose. The Loupe used for this microscope functions like a magnifying glass and approximate ray diagrams can be drawn, but the loupe barrel contains two convex lenses: a field lens under an upper eyepiece lens. The two lenses are separated by about 2 cm and are thinner (less fat) than a single magnifying glass of the same focal length. The loupe has less image distortion. Hooke's microscopes in the 1660's were made with single objective and eyepiece lenses and had considerable image distortion. He drew what he saw carefully one part at time and *compiled* his drawings, compensating for the distortions.