Reflections and Rotations on the Cartesian plane

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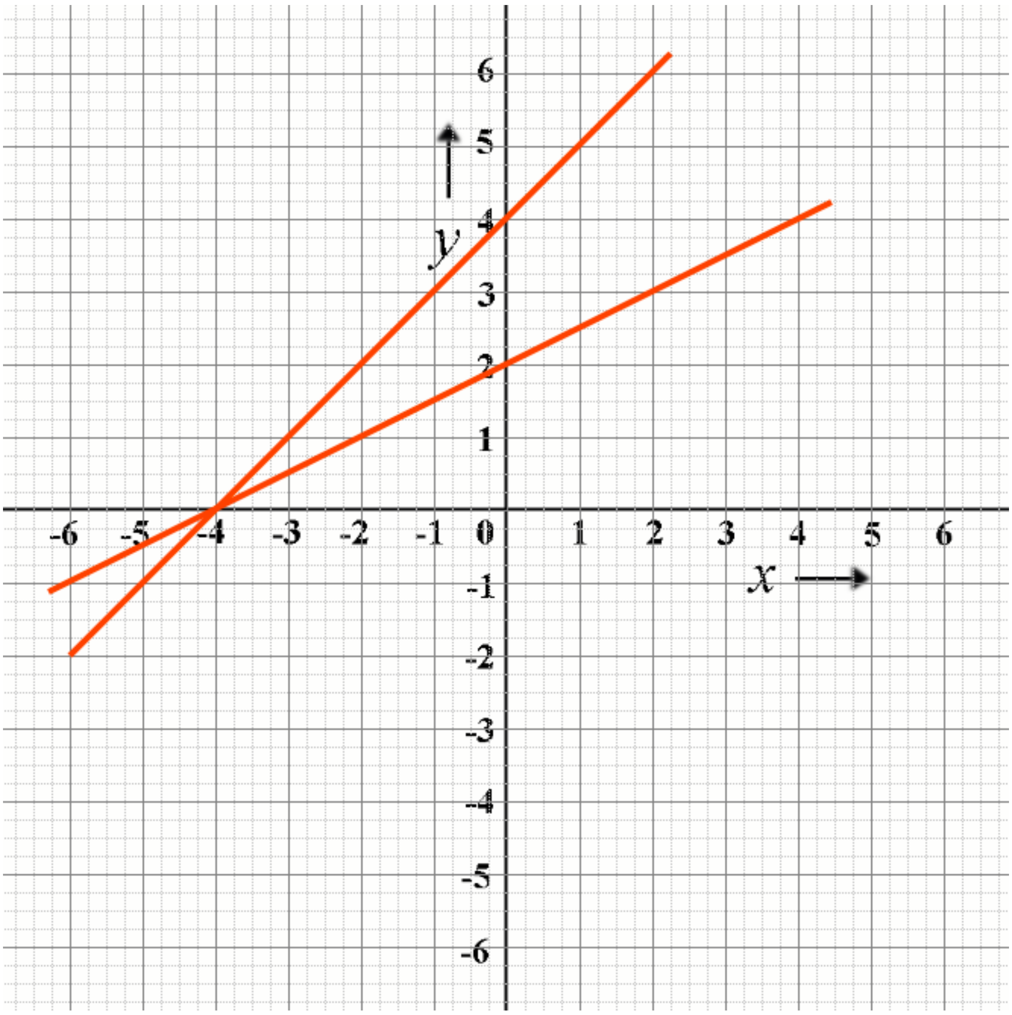
If you read *Transformations* [1] in the mathematics index you will see how to modify equations in x and y to translate and scale figures on the Cartesian plane. In this article we see how to reflect lines and circles in mirror lines made by the x and y axes and how to rotate figures by 90 degrees.

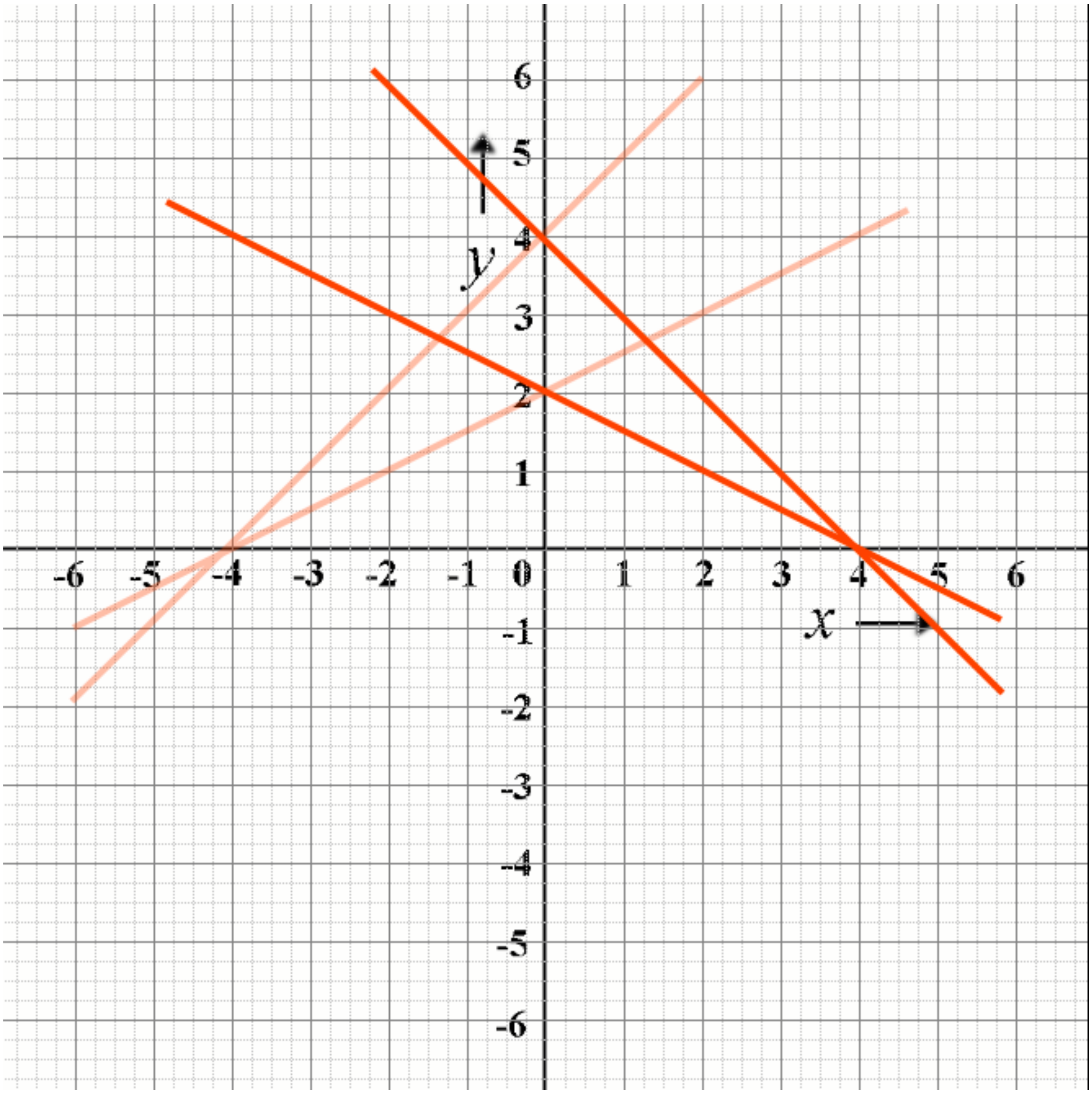
Two lines are drawn on the plane.



Their equations are … $y = x + 4$ …. [1]

… and … $y = {}^{1}\!/_{2}\, x + 2$ …. [2]

Reflection in the *y* axis



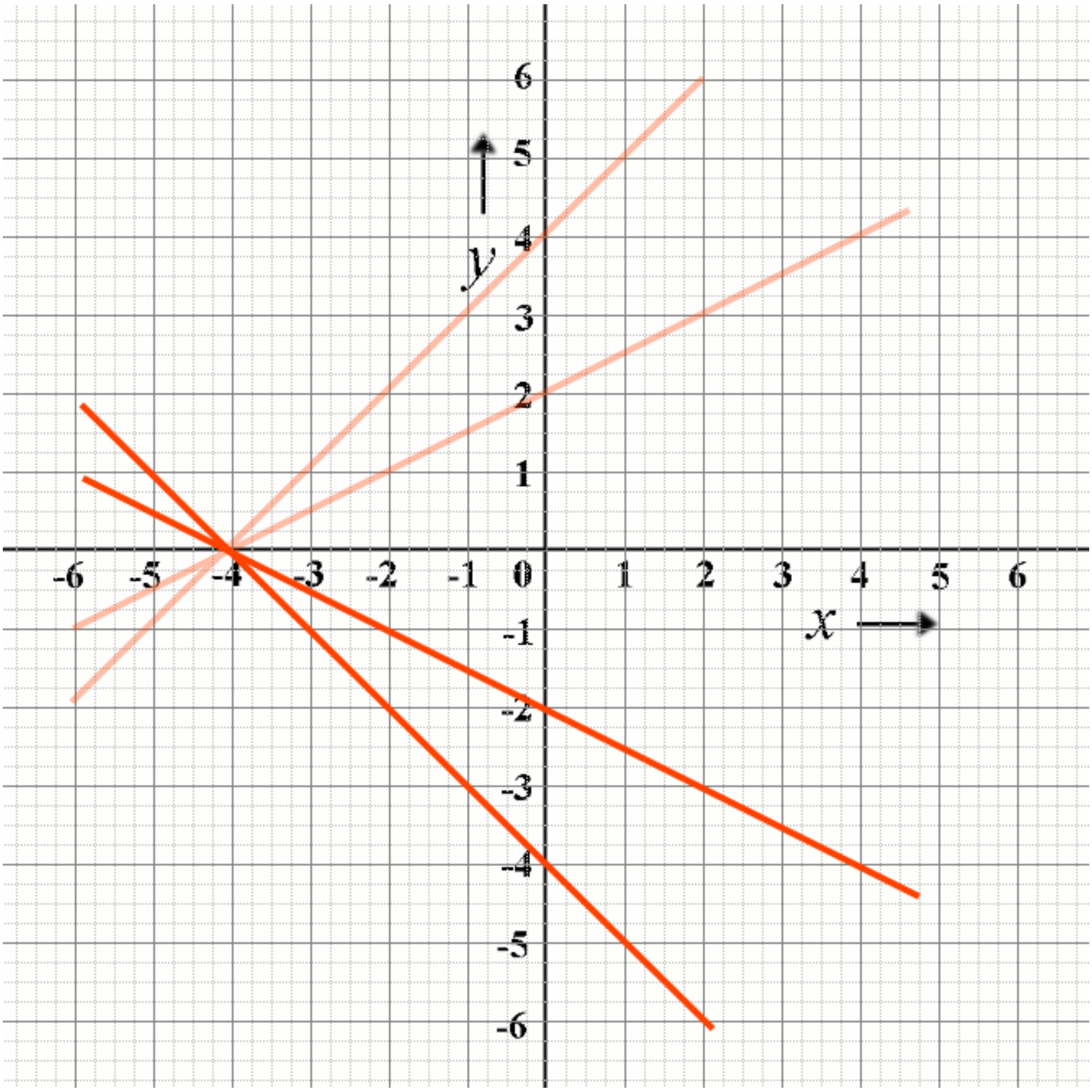
The original lines (in pale red) are reflected in the *y* axis ($x = 0$) by replacing *x* with -*x* in equations 1 and 2.

$$y = -x + 4$$

$$y = -{}^{1}/_{2}\,x + 2$$

Note that the *x* values of all points on any line are converted to -*x,* except for points on the line $x = 0$, which is the mirror line.

Reflection in the *x* axis



The original lines in pale red are reflected in the *x* axis ($y = 0$) by replacing y with $-y$ in equations 1 and 2.

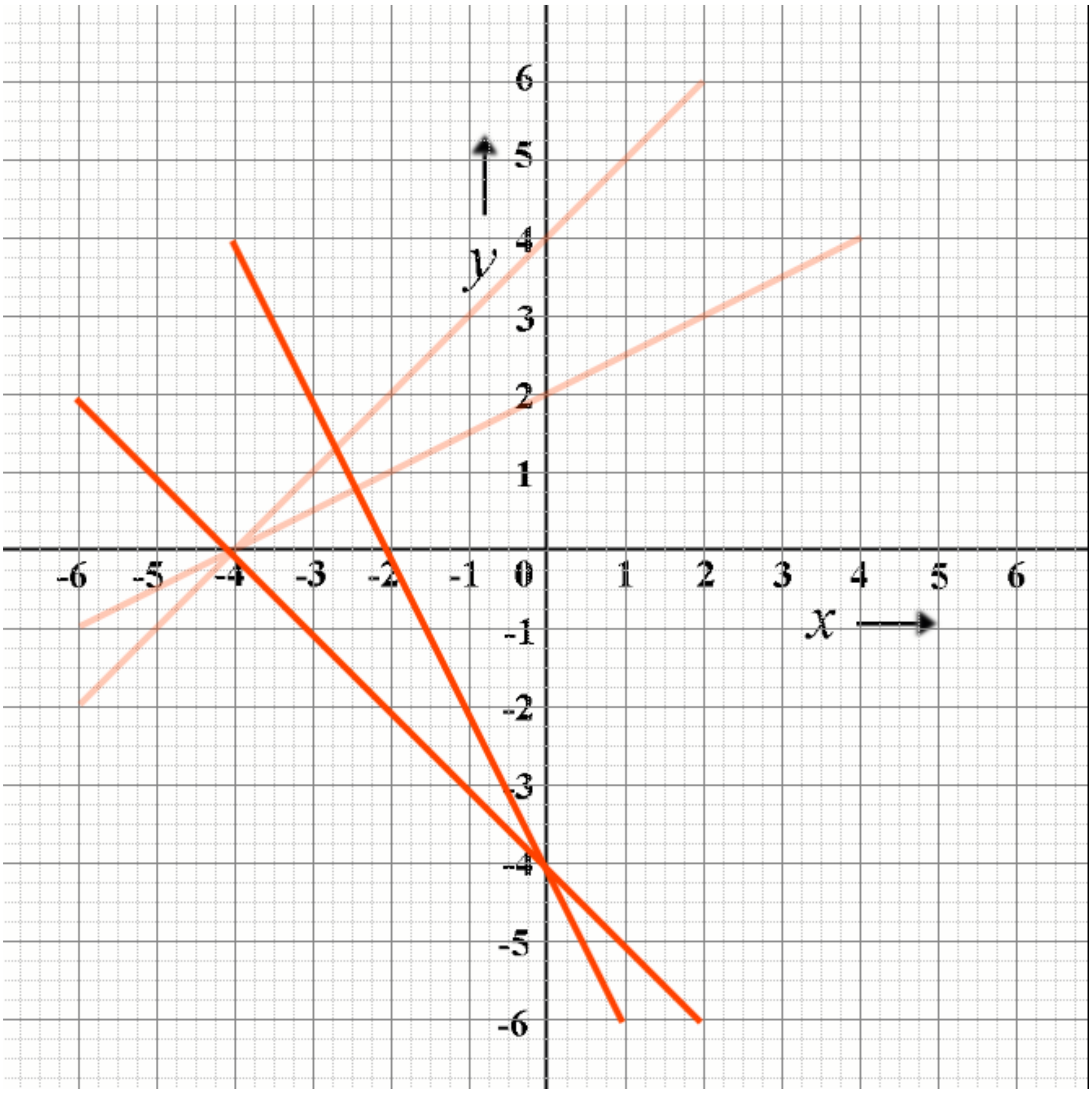
$$-y = x + 4$$

Modified equation … $y = -x - 3$

$$-y = {}^{1}/_{2}\, x + 2$$

Modified equation … $y = -{}^{1}/_{2}\, x - 2$

Positive rotation about (0, 0) by 90º



The original lines in pale red are rotated by +90º (or -270º) by exchanging x <=> y and replacing x with $-x$ in equations 1 and 2.

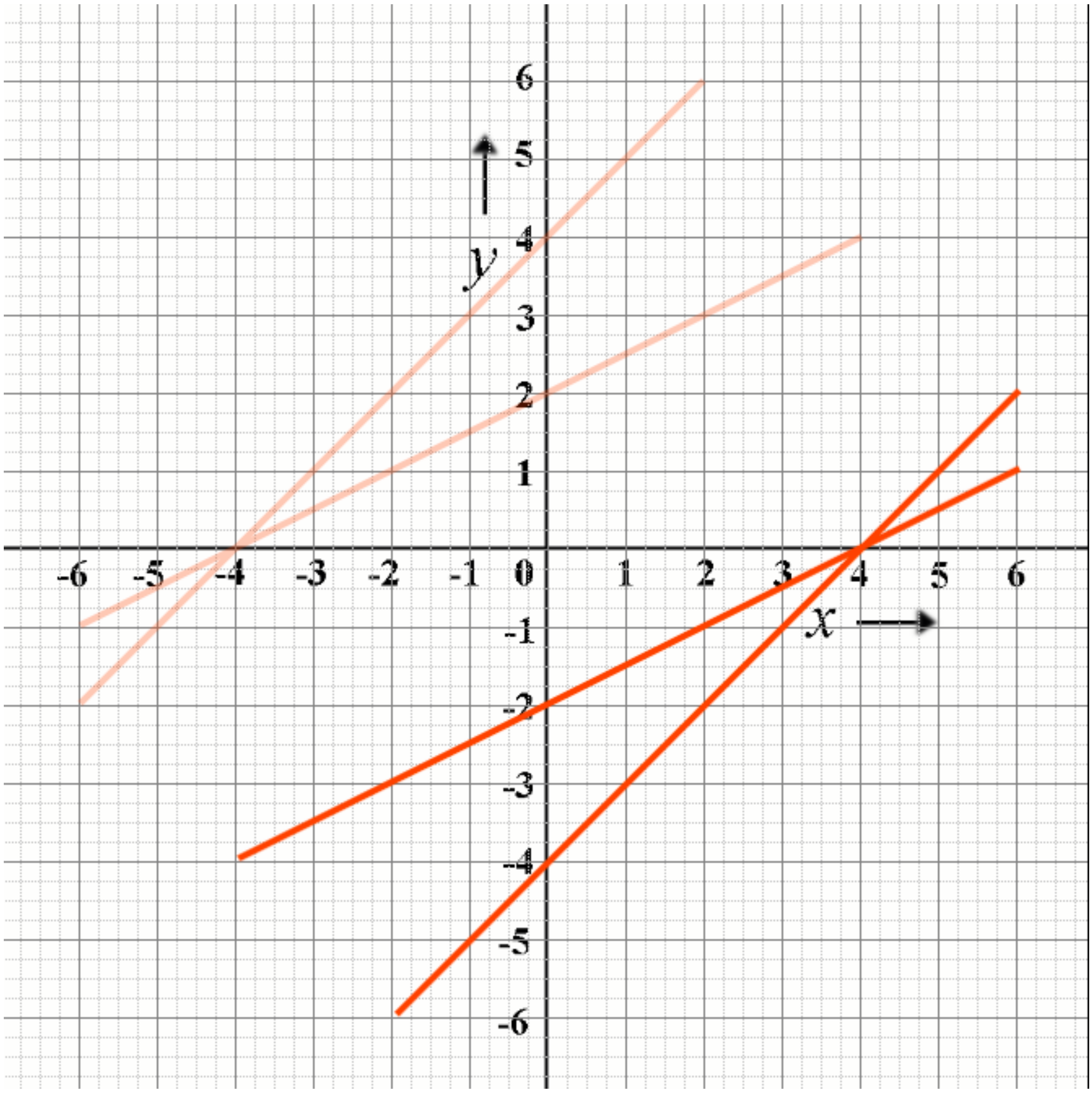
$$-x = y + 4$$

Modified equation … $y = -x - 4$

$$-x = \tfrac{1}{2}y + 2$$

Modified equation … $y = -2x - 4$

Positive rotation about (0, 0) by 180º



The original lines (in pale red) are rotated 180º by replacing ***x*** with ***-x*** and ***y*** with ***-y*** in equations 1 and 2.

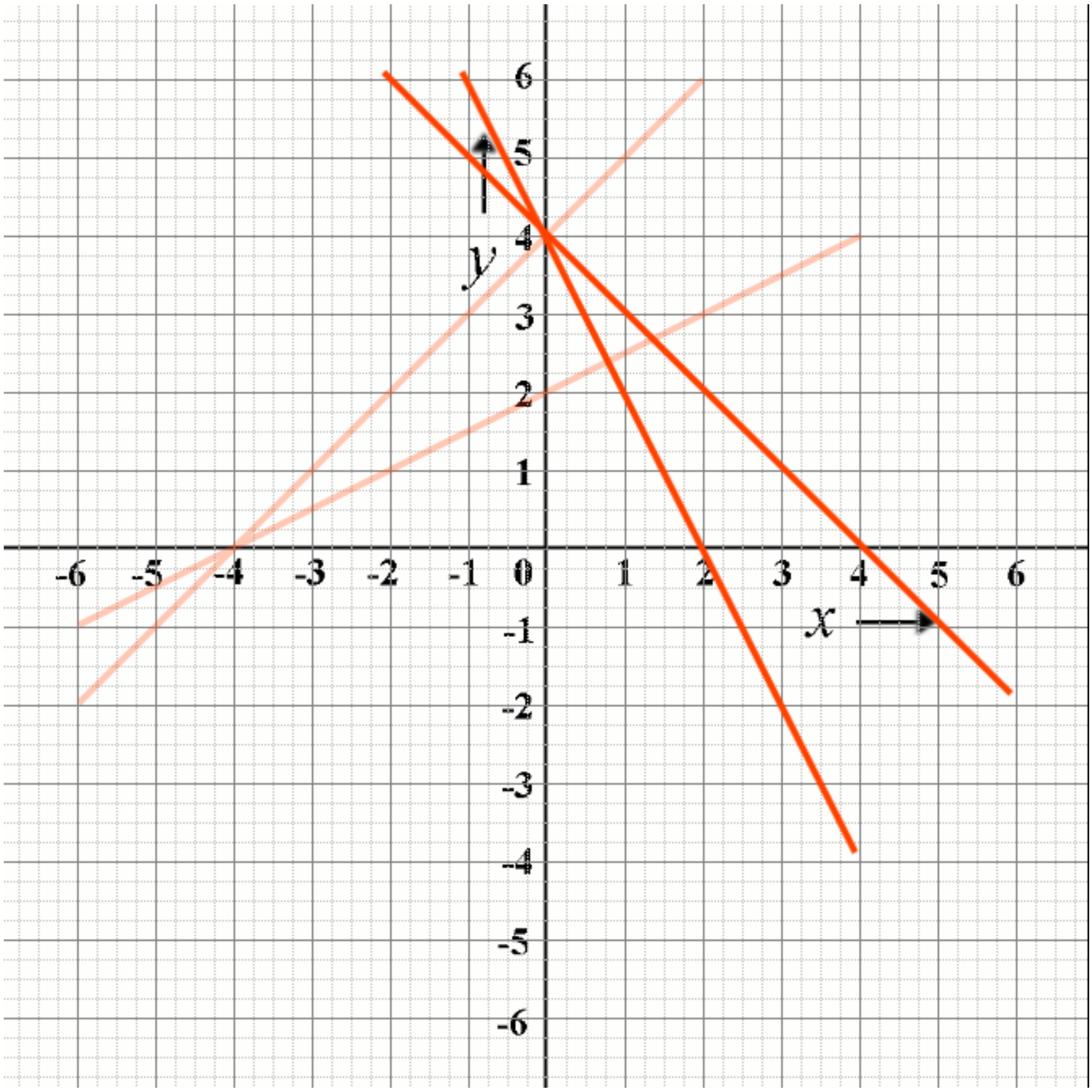
$$-y = -x + 4$$

Modified equation … $y = x - 4$

$$-y = -\,{}^{1}/_{2}\,x + 2$$

Modified equation … $y = {}^{1}/_{2}\,x - 2$

Positive rotation about (0, 0) by 270º



The original lines in pale red are rotated by +270º (or -90º) by exchanging $x \Leftrightarrow y$ and replacing y with $-y$ in equations 1 and 2.

$$x = -y + 4$$

Modified equation … $y = -x + 4$

$$x = -\frac{1}{2}y + 2$$

Modified equation … $y = -2x + 4$

The following transformations have been used.

1 The transformation $x \rightarrow -x$ reflects lines in the y axis, which is the line $x = 0$ which is left undisturbed.

2 The transformation $y \rightarrow -y$ reflects lines in the x axis, which is the line $y = 0$ which is left undisturbed.

3 Both transformations: $x \rightarrow -x$ and $y \rightarrow -y$ rotates lines 180º about the origin (0, 0), which is the only point on the plane left undisturbed. A rotation of 180º is equivalent to refection in the y axis followed by reflection in the x axis.

4 Exchanging $x \Leftrightarrow y$ and transforming $x \rightarrow -x$ rotates lines about the origin by +90º (or -270º).

5 Exchanging $x \Leftrightarrow y$ and transforming $y \rightarrow -y$ rotates lines about the origin by -270º (or +90º).

Note : the first three transformations are easily understood but the details of 4 and 5 may be difficult to remember. They can be recalled with a simple example, for instance by exchanging $x \Leftrightarrow y$ and rotating the line $y = x + 1$, if they are needed and have been forgotten.

Two possible transformations remain.

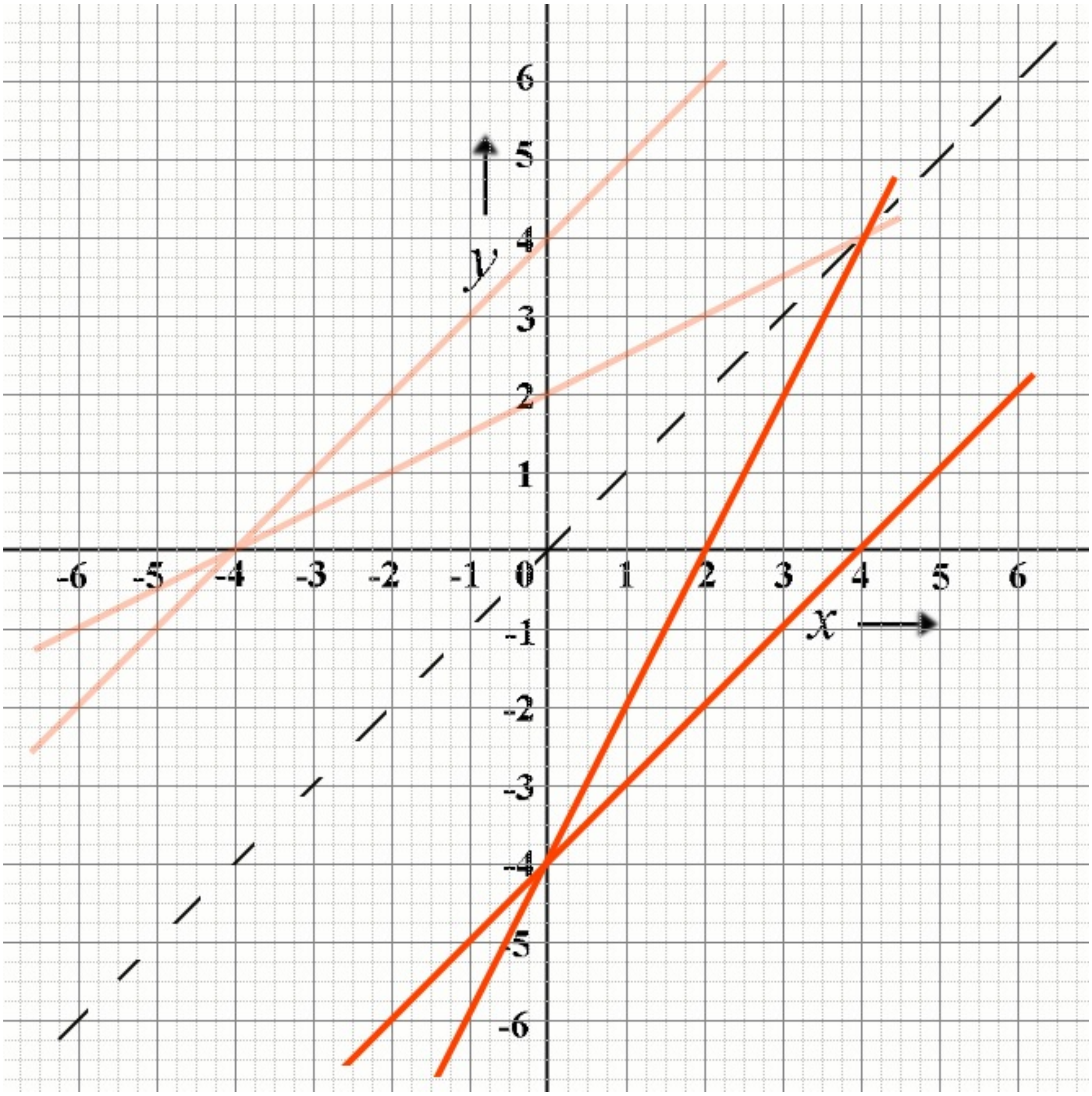
6 Exchanging $x \Leftrightarrow y$ ... leaving both signs unchanged.

7 Exchanging $x \Leftrightarrow y$ …and transforming $x \rightarrow -x$ and $y \rightarrow -y$.

Along with leaving x and y unchanged, which is a rotation of 360º about the origin, options 6 and 7 complete the eight possible permutations of x and y with positive or negative signs.

The effects of 6 and 7 are shown below.

Reflection in the line $y = x$



The original lines (in pale red) are reflected in the line $y = x$ by replacing y with x and x with y in equations 1 and 2.

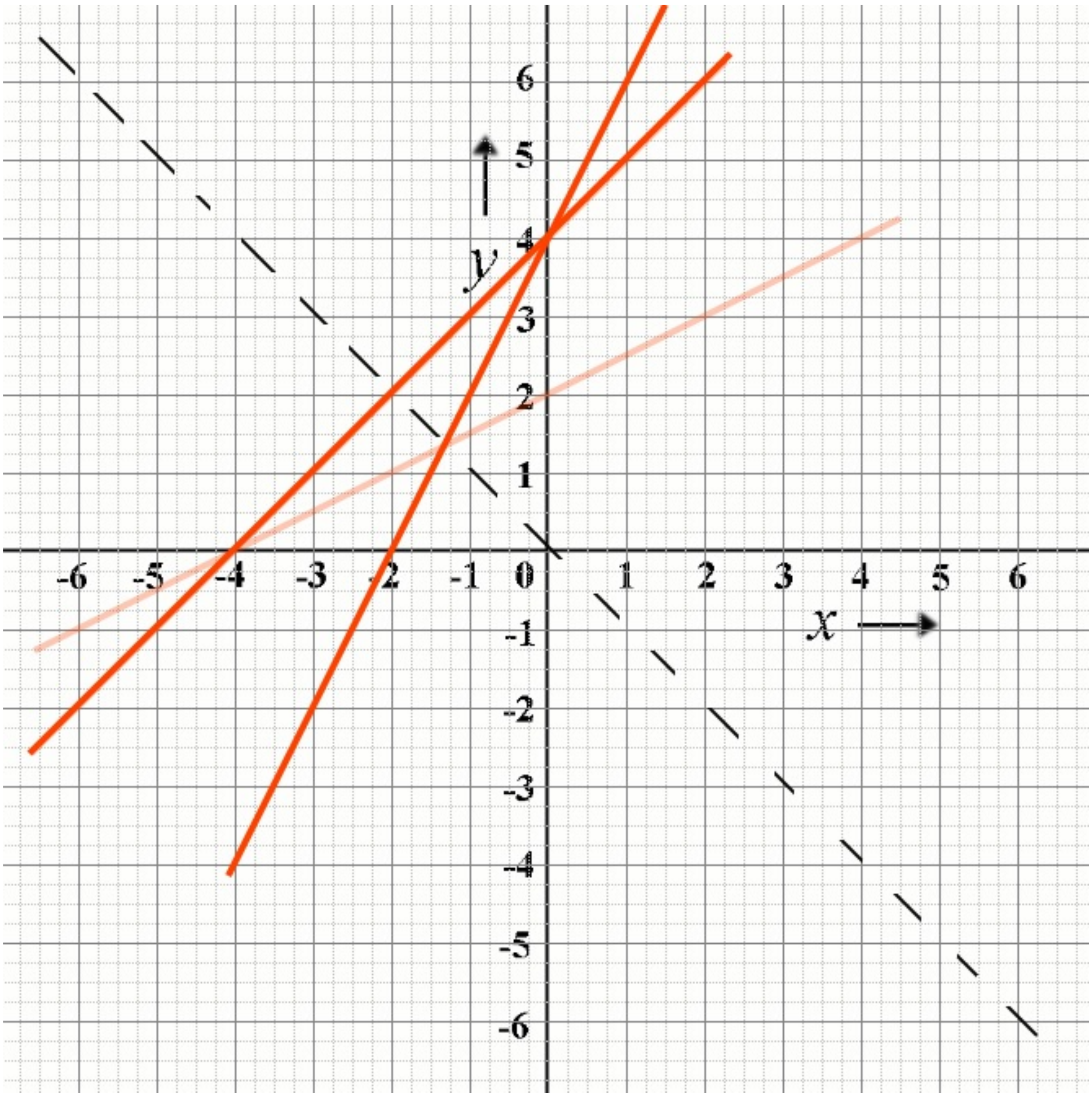
$$x = y + 4$$

Modified equation … $y = x - 4$

$$x = {}^{1}/_{2}\, y + 2$$

Modified equation … $y = 2x - 4$

Reflection in the line $y = -x$



The original lines (in pale red) are reflected in the line $y = -x$ by replacing y with $-x$ and x with $-y$ in equations 1 and 2.

$$-x = -y + 4$$

Modified equation … $y = x + 4$

$$-x = -\tfrac{1}{2}y + 2$$

Modified equation … $y = 2x + 4$