Translation and Enlargement on the Cartesian plane

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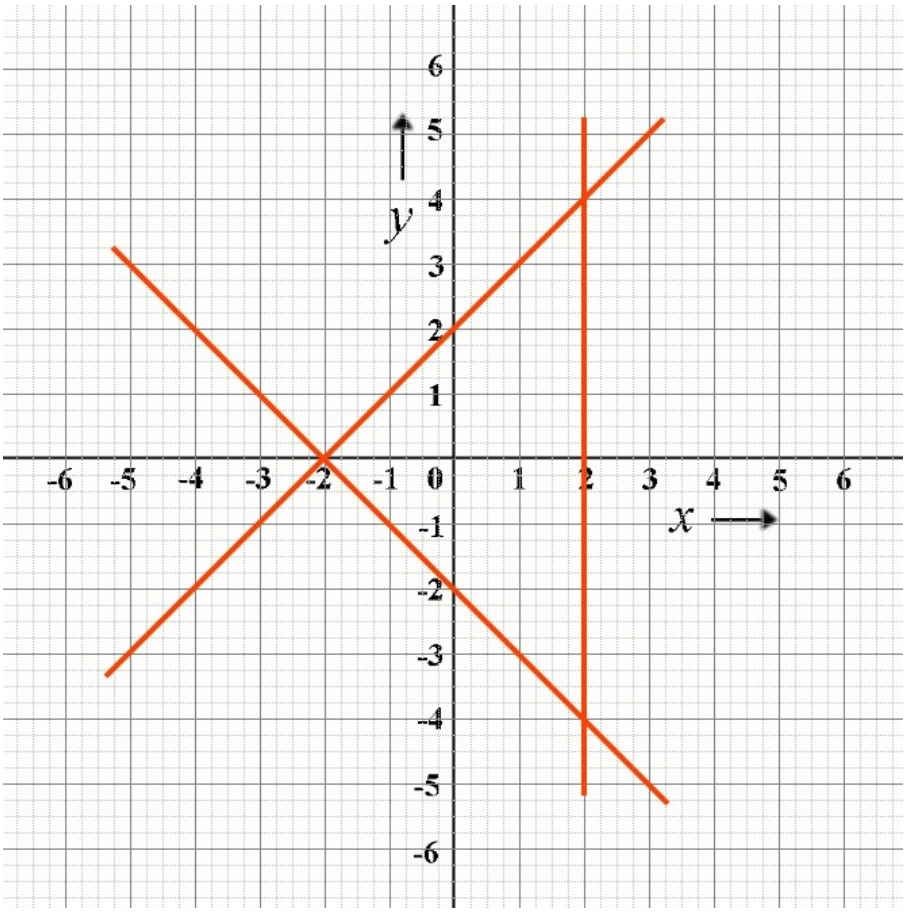
The *Cartesian plane* that connected algebra with geometry was Descartes clever contribution to mathematics. Connection between equations (algebra) and their graphs (geometry) can be shown by making substitutions in equations to reflect and rotate the lines that represent them on the plane.



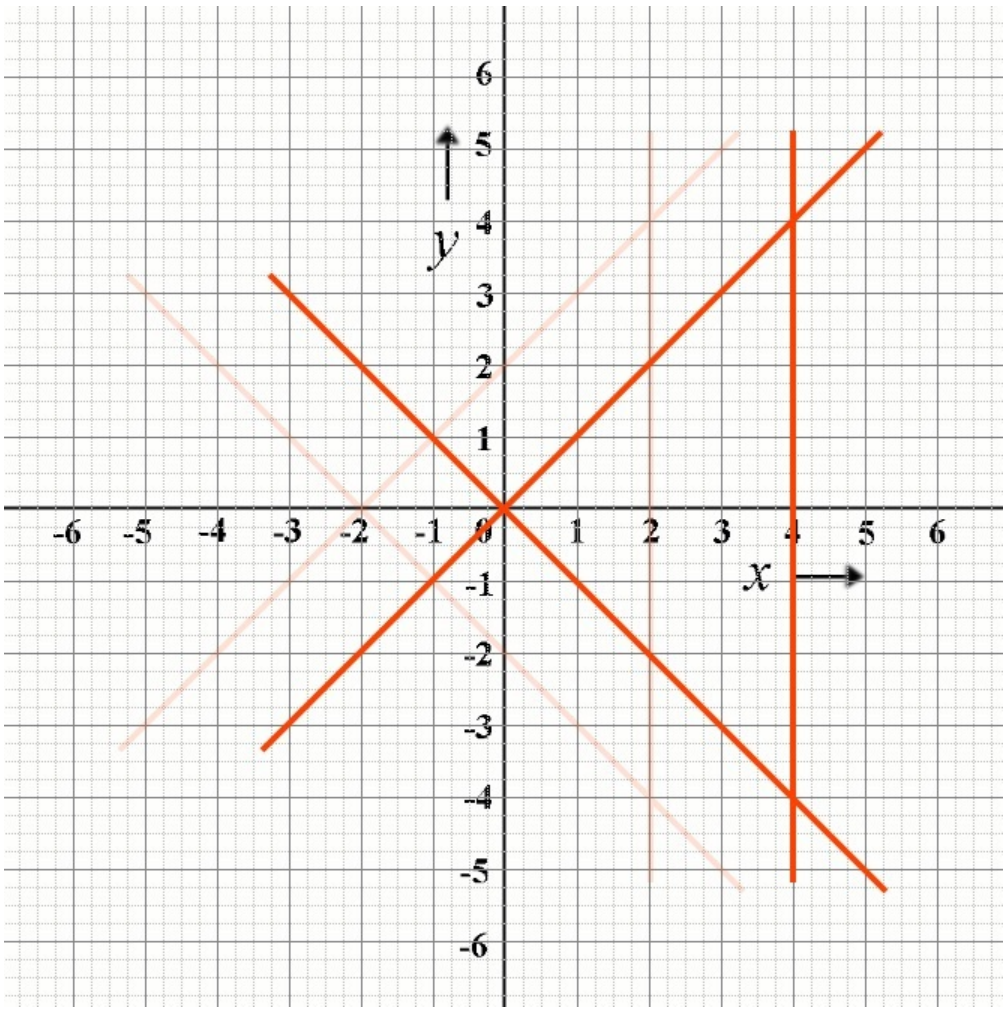
Three lines are drawn on the plane …

$$y = x + 2 \qquad \ldots [1]$$

$$y = -x - 2 \qquad \ldots [2]$$

$$x = 2 \qquad \ldots [3]$$

Translation in the positive *x* direction



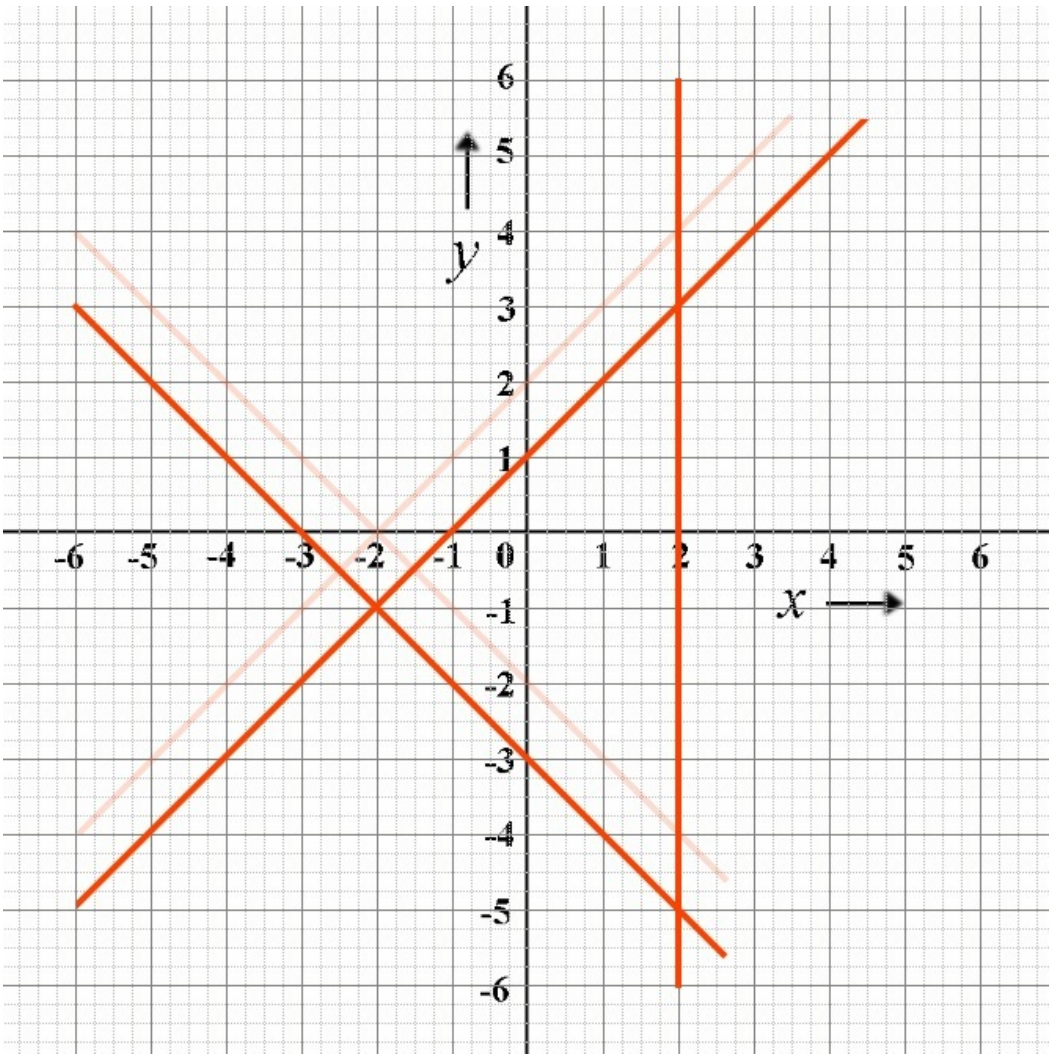
Lines are moved two units in the **positive** *x* direction by replacing *x* by (*x* - 2) in equations 1, 2 and 3.

$$y = (x - 2) + 2$$
$$y = -(x - 2) - 2$$
$$(x - 2) = 2$$

The modified equations are …

$$y = x$$
$$y = -x$$
$$x = 4$$

Translation in the negative *y* direction



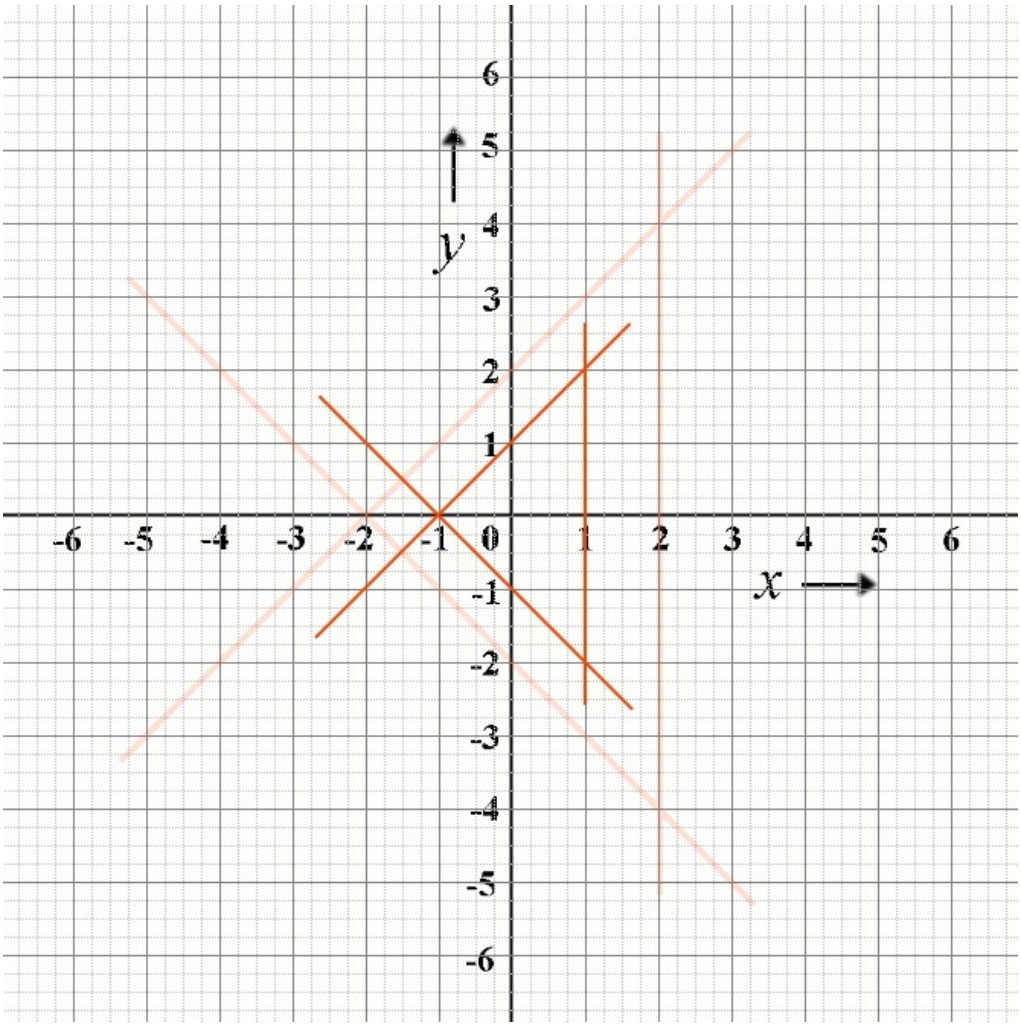
The lines are moved one unit in the **negative** *y* direction by replacing *y* by $(y + 1)$ in equations 1, 2 and 3.

$$(y + 1) = x + 2$$
$$(y + 1) = -x - 2$$
$$x = 2$$

The modified equations are …

$$y = x + 1$$
$$y = -x - 3$$
$$x = 2$$

Scaling: enlargement or reduction



The enclosed triangle is reduced to half size by replacing x by $2x$ and y by $2y$.

$$2y = 2x + 2$$
$$2y = -2x - 2$$
$$2x = 2$$

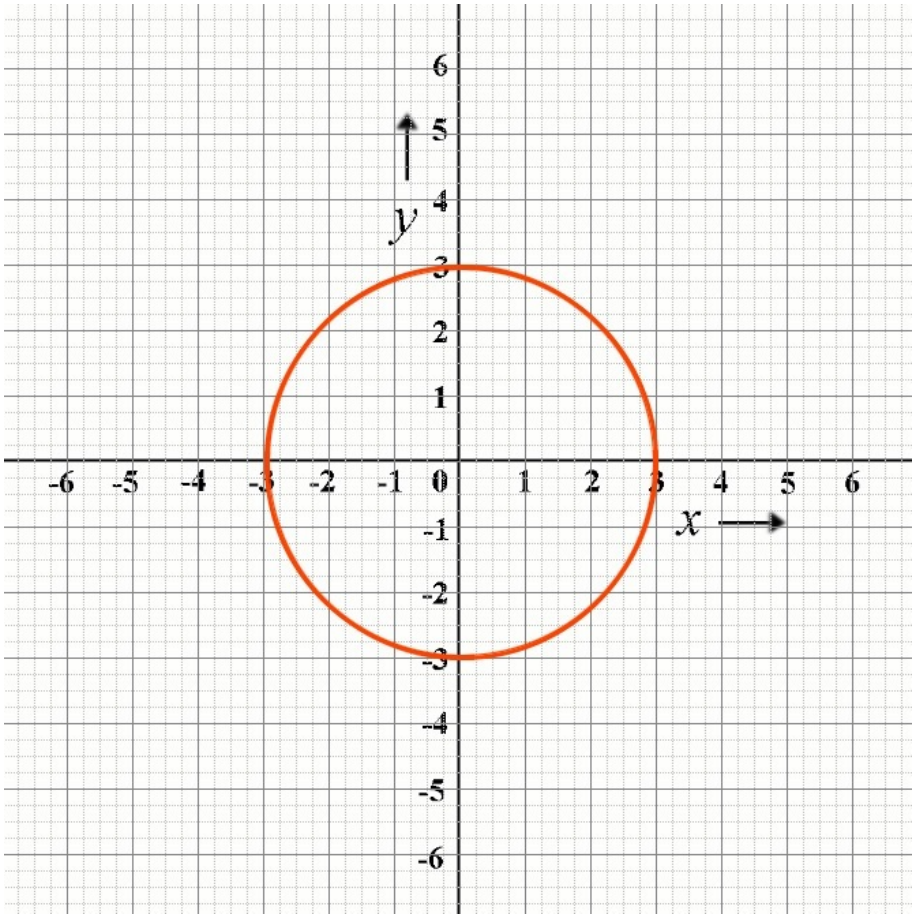
The modified equations are …

$$y = x + 1$$
$$y = -x - 1$$
$$x = 1$$

Circles

The equation of a circle of radius a centred on the origin (0, 0) is …

$$x^2 + y^2 = a^2$$



The equation of the circle of radius 3 is …

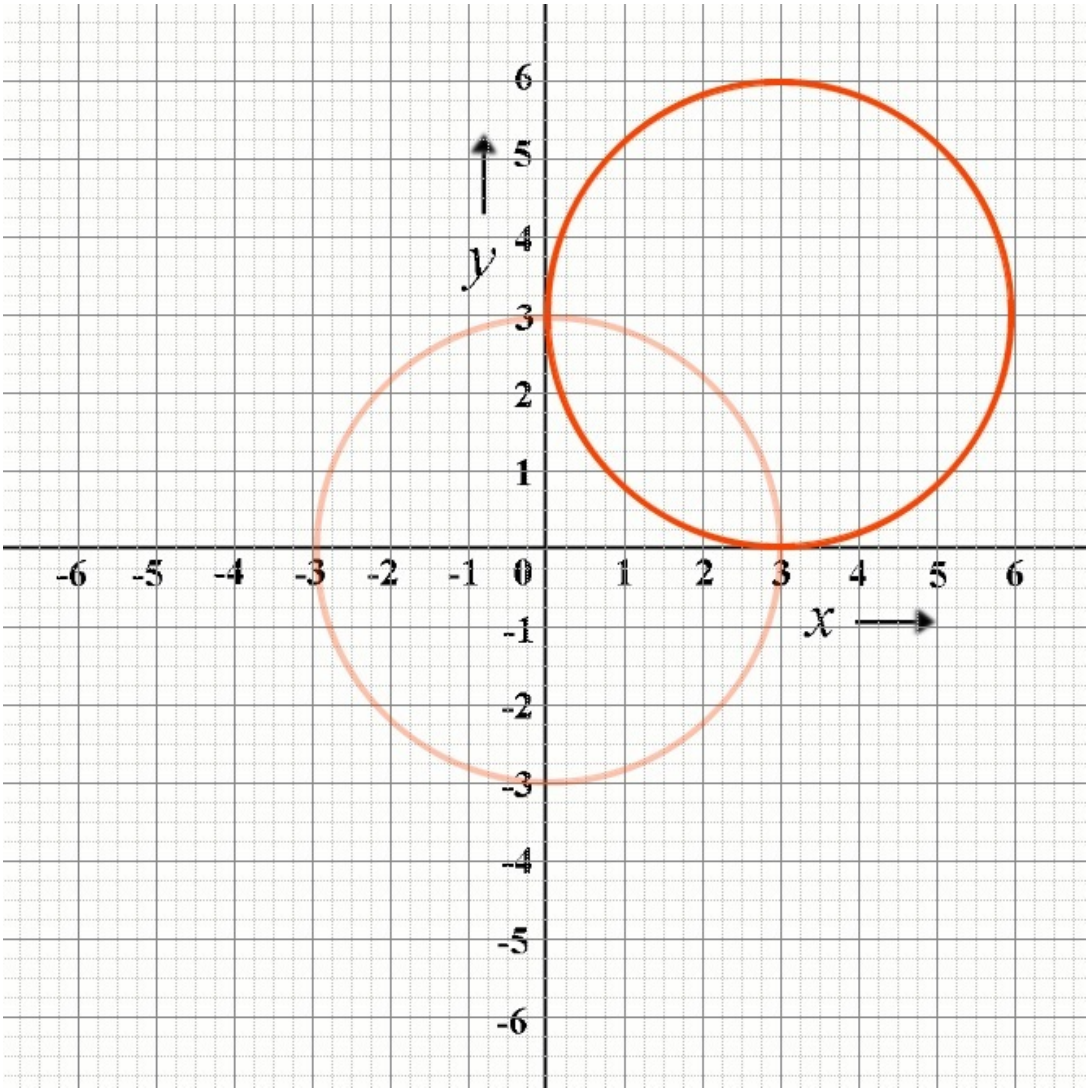
$$x^2 + y^2 = 9$$

The points (-3, 0), (0, 3), (3, 0) and (0, -3) lie on the circle where it crosses the axes.

If x lies between -3 and +3, y is given by $\pm\sqrt{(9 - x^2)}$. If x is outside this range $(9 - x^2)$ is negative and real values for y cannot be found.

Translation

A circle is moved on the plane as above. .



The pink circle is translated 3 units in the positive x direction, and three units in the positive y direction, by replacing x by $(x - 3)$ and y by $(y - 3)$ in the equation of the circle …

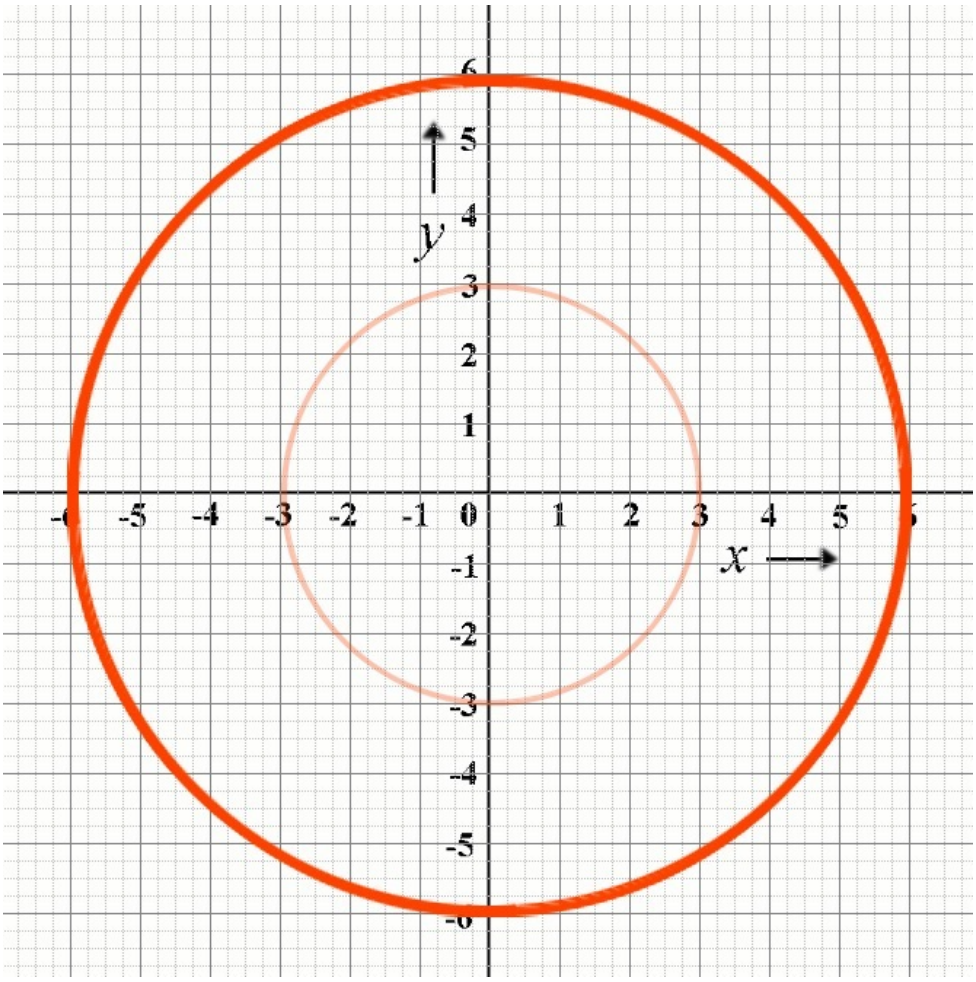
$$x^2 + y^2 = 9$$

The modified equation is …

$$(x - 3)^2 + (y - 3)^2 = 9$$

Scaling: enlargement or reduction

Scaling on the plane is achieved as above.



The circle of radius 3 is doubled in size by replacing x by $x/2$ and y by $y/2$ in the equation of the circle …

$$x^2 + y^2 = 9$$

The modified equations is …

$$(x/2)^2 + (y/2)^2 = 9$$

… which simplifies to … $x^2 + y^2 = 36$