Pascal's Paradox

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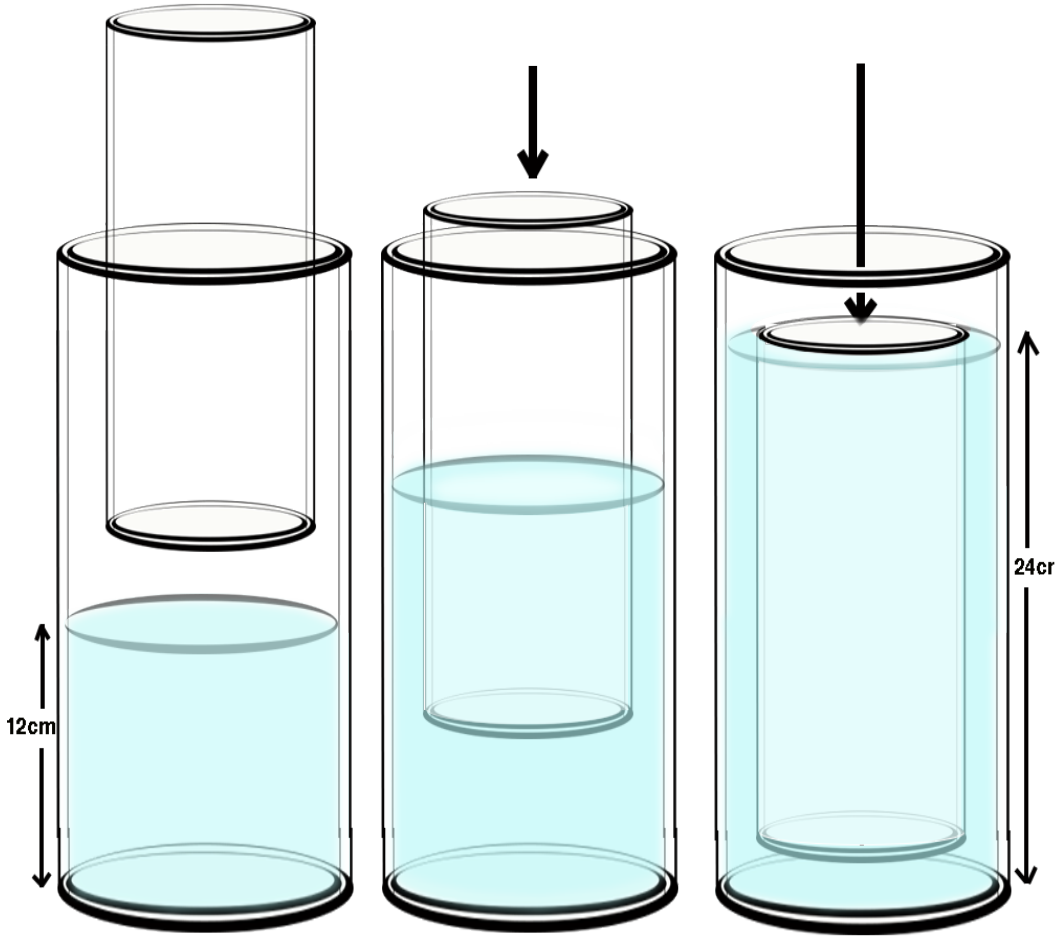
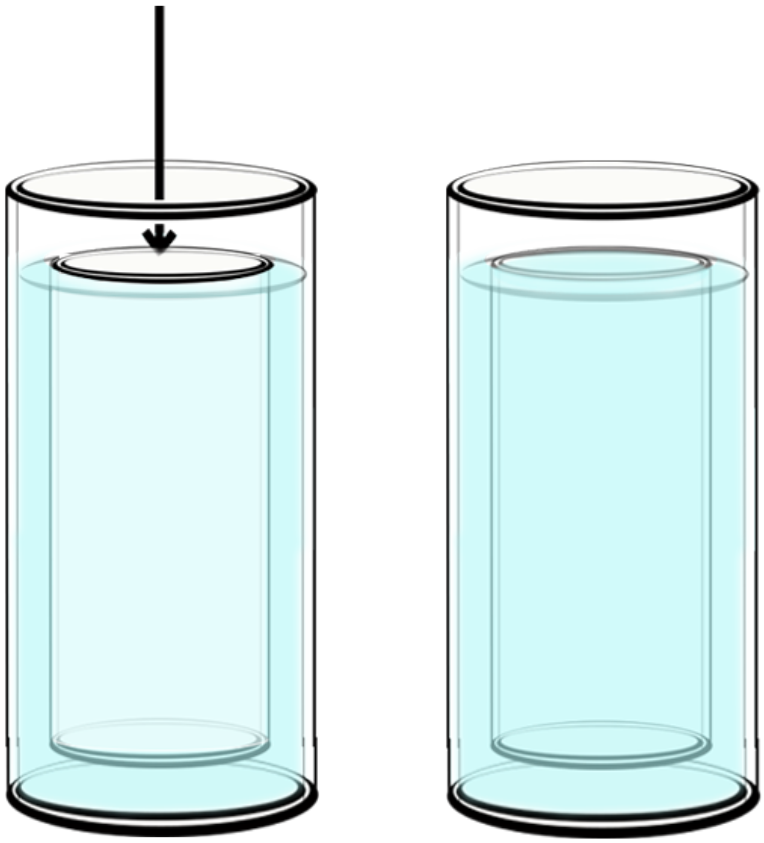
When a set of scales is put under each cylinder in the diagram below, the scale reading on the left (the weight of two litres of water, assuming the containers are light) increases as the empty two litre cylinder is pushed down until, on the right, the scale reading has increased to the weight of four litres of water.



Only two litres of water are in the cylinder on the right but the scale reads the weight of four litres (see below).



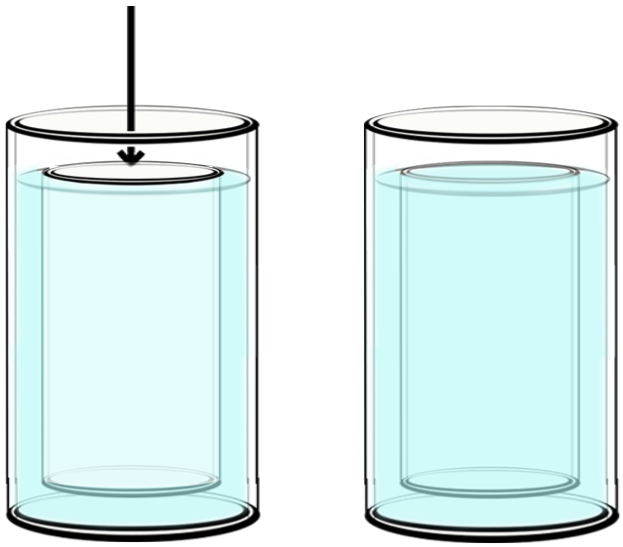
Gradually filling the empty cylinder with water reduces the applied downward force that was needed to balance the buoyancy force. The balance reading remains the same until the cylinder is full and the reading then clearly is the weight of four litres of water.

The balance reads the force of the water on the base of the outer cylinder. In terms of pressure ρgh this force F is given by …

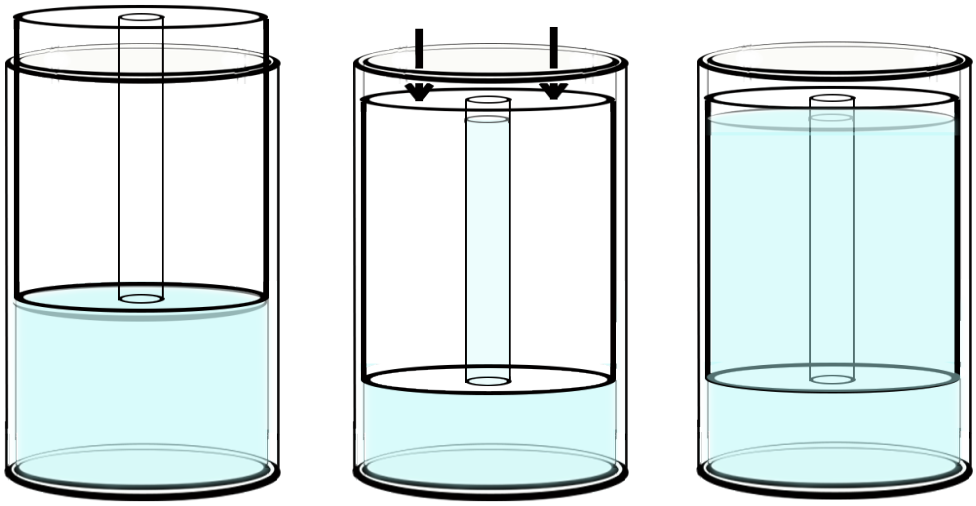
$$F = \rho gh\_A$$

The height h in this equation for the force on the base of area A (the *effective weight* of the water) is the maximum height of water in the container, not the average height. This means that the force on the base can be greater than the *actual weight* of water contained. This curious result is called Pascal's Paradox.

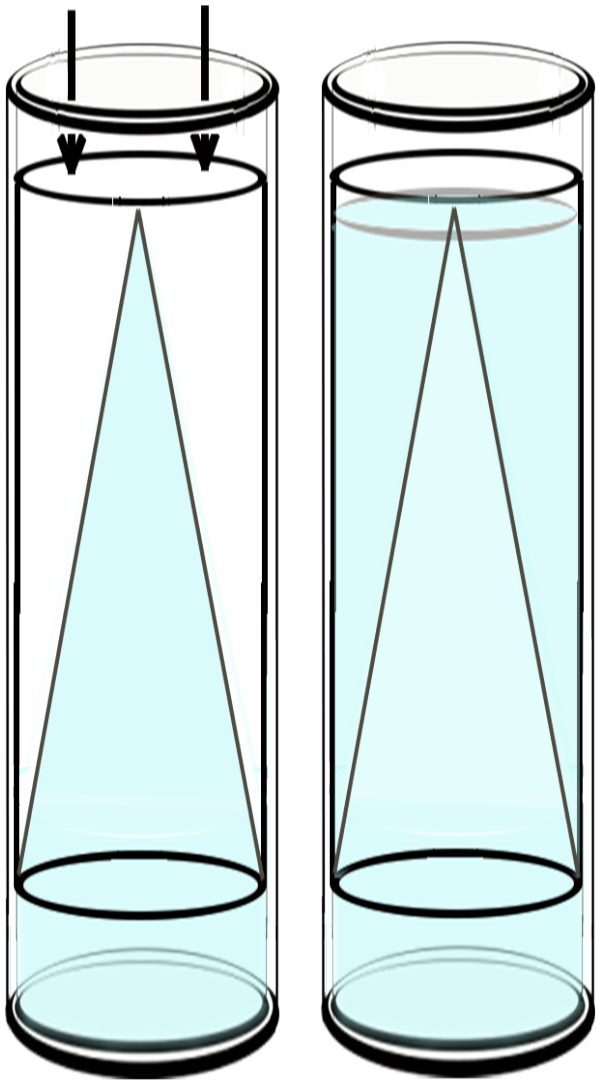
Examples



1 An outer container holds water. The inner container is empty on the left and held down in the water. On the right the inner container is full of water. *Explain clearly why the force on the base of the outer container is the same on the left and right.*



2 An outer container holds water. An empty inner container with an open hollow centre is held down in the water in the middle diagram. On the right this inner container has been filled with water. The water rises to twice its height in the centre and on the right. By how much is the force of water pressing down on the base of the outer container increased from left, centre, to right?



3 An empty cylinder with an insert in the form of a hollow cone has been forced down and held into the container on the left. As this was done the empty cone was filled with water as air escaped through a hole at the tip of the cone (not shown). The empty shape is then filled with water to the height of the tip of the cone.

a Explain, using the concept of pressure, why the force on the base of the outer cylinder is the same on the left and on the right.

b How may a well known paradox be resolved with this demonstration?