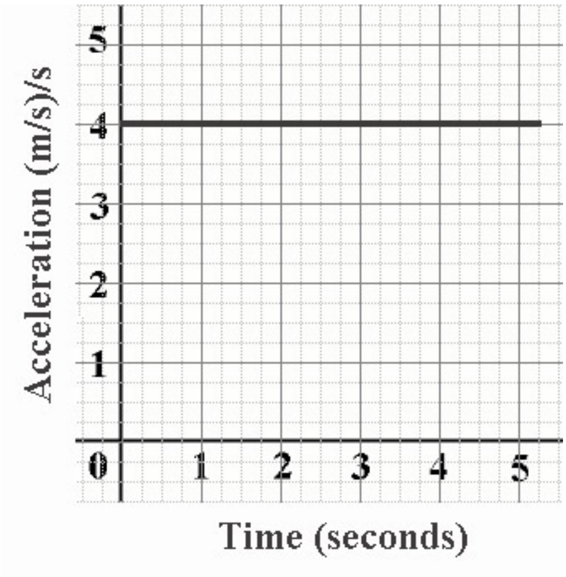
Constant acceleration

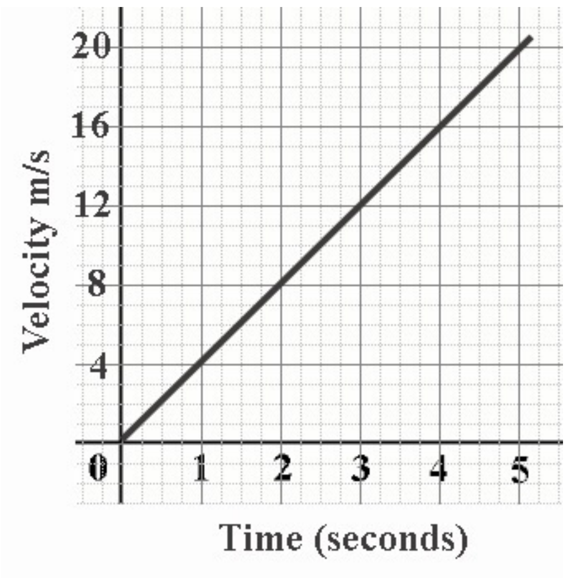
Shannon and Ian Jacobs

We simplify calculations of velocity and displacement when heavy objects fall, are thrown, or roll on hills, by imagining that the force due to gravity is vertical and the same everywhere. *The acceleration is then constant*.

Question: acceleration is constant with a value of +4 (m/s)/s in a straight line. Describe the velocity as a function of time.



Answer: the velocity increases by 4 m/s each second.



The velocity/time plot is a straight line with a slope of 4 (m/s)/s.

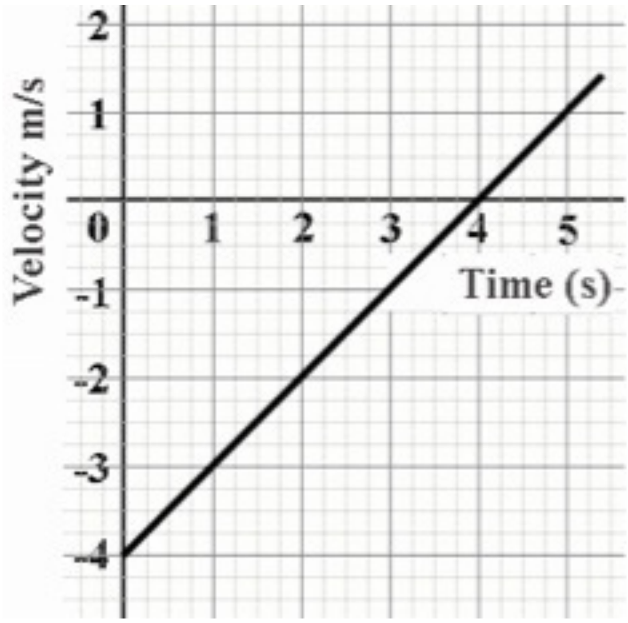
On the (x, y) plane the equation of the line on the graph above would be …

$$y = mx \text{ … where } m \text{ is the slope of the line.}$$

Changing variables: on the (t, v) plane the equation of the line is …

$$v = at \text{ … where } a \text{ is the slope of the line.}$$

Question: what would the velocity have been at 4 seconds, if the velocity at zero time was -4 m/s?



Answer: *from what we know we sketch the velocity-time graph.*

The velocity at zero time was -4 m/s and the slope is 1 (m/s)/s.

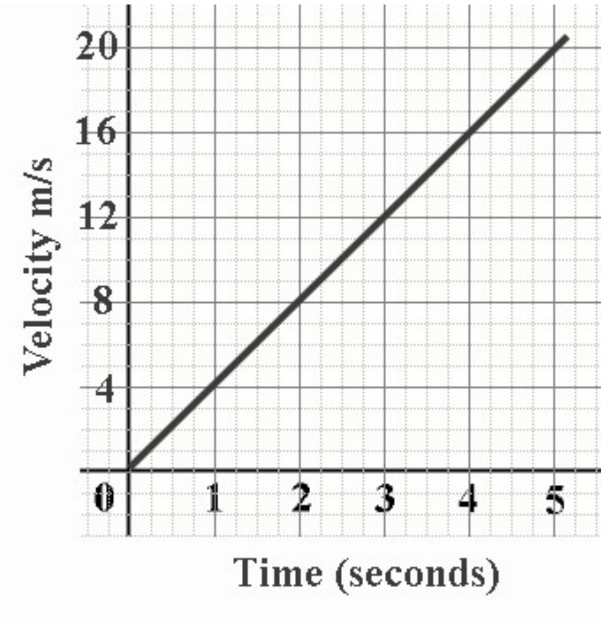
The equation of the line is … $v = t - 4$

Using the common symbol for initial velocity "u" the equation is …

$$v = at + u$$

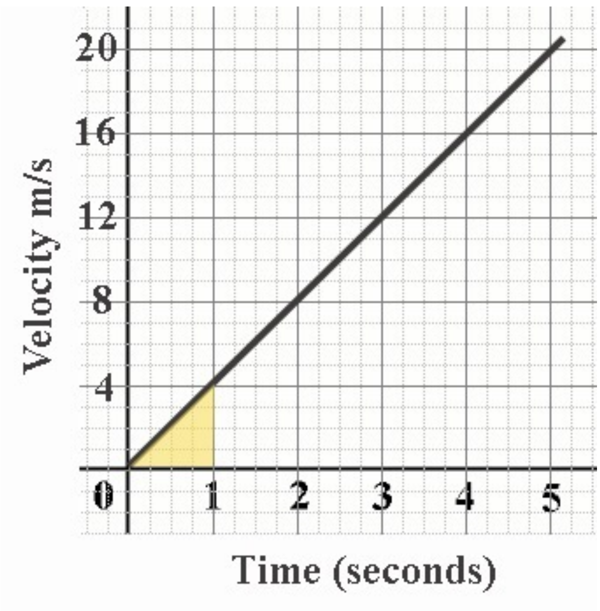
Note: v, u … and/or … a may be positive or negative.

We are given this velocity-time graph



We know *at once* that the acceleration is constant *because the line is straight*, and that acceleration in (m/s)/s is *the slope of the line*.

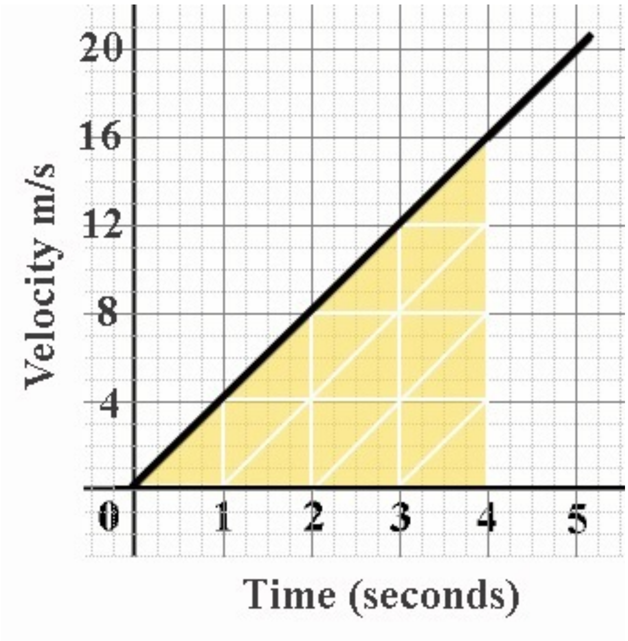
Question: given the velocity-time graph above, what is the *displacement* of the object from zero at one second?



Answer: the area under the velocity-time graph is the displacement in *metres* because velocity is in m/s and time is in seconds.

At one second the object is *two metres* from $(0, 0)$.

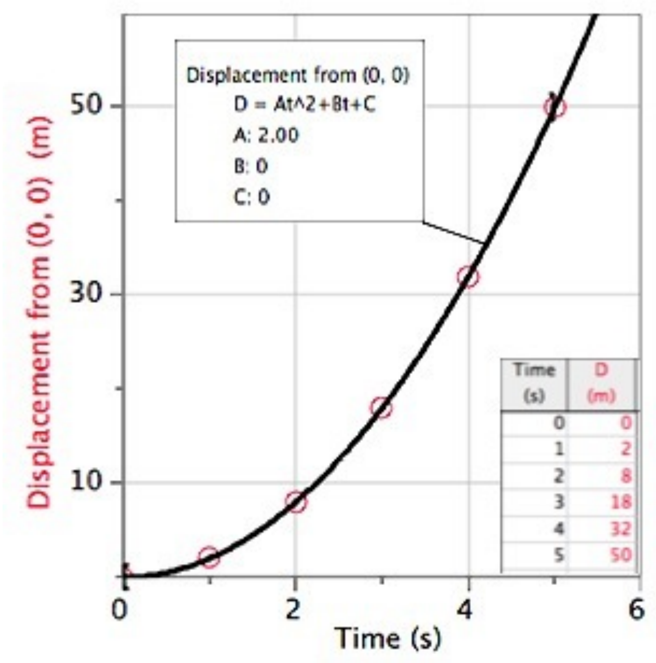
At any time “t” the displacement is the area under a velocity-time plot.



The area of the yellow triangle is … $^1/_2$ x 4 x 16 = 32 metres.

At four seconds the object is 32 metres from (0, 0).

The straight line graphs above were drawn by hand, but the displacement graph is best drawn with computer calculation of the curve of best fit.



The curve of best fit is a parabola, a quadratic function in t squared:

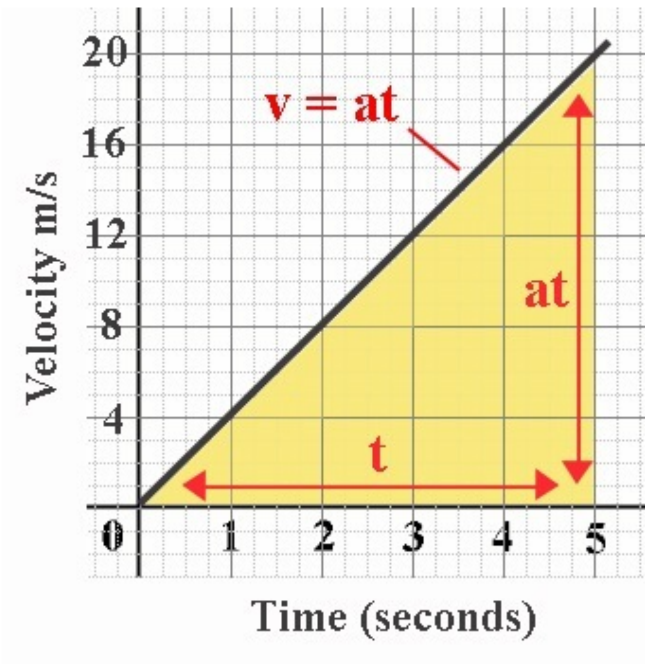
The equation of the curve on the graph above describes a *parabola*.

$$D = At^2 + Bt + C$$

… where, because the parabola passes through (0, 0) at the lowest point, B and C are zero.

Notice that the value of A is +2, which is half the acceleration, +4 (m/s)/s.

To understand why the coefficient of t^2 is $^a/_2$ remember that the area of a triangle is *half* the base times the height. The displacement values (listed in the table on the graph above) are found from the areas of triangles on the velocity-time graph.



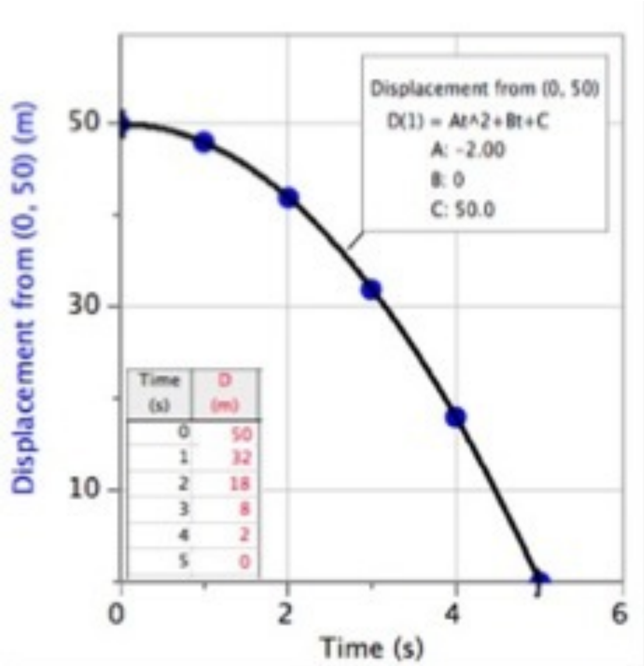
The equation of the line is $v = at$. The height of the triangle is v (which equals at) and the base is t. Area on the (t, v) plane is in metres. The area of the triangle gives the displacement from (0, 0) at time t, which is …

$$^1/_2\, at^2 = [^1/_2\, a]t^2$$

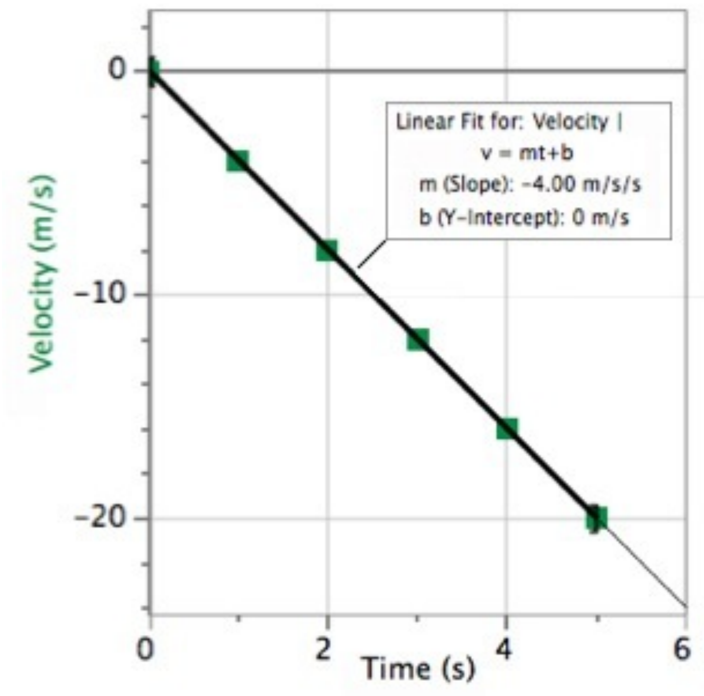
The coefficient of t^2 in the equation for the displacement as a function of time is *half the acceleration*. The half comes from the area of the triangle.

A falling object: clever reasoning

Approximate data for a stone falling from rest on Mars is listed and plotted below.



The acceleration is a *constant* -4 (m/s)/s …(twice the coefficient of t^2).



Because the stone fell from rest at zero time, the velocity-time graph is a straight line from (0, 0), with a slope of -4 (m/s)/s.