Solving equations

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Information from text (English) is translated into mathematical statements (equations) that can be solved for unknowns. Two equations are required if there are two unknowns.

Here is a typical question.

Pencils are cheaper than pens. The difference is 30 Baht. One pen and two pencils cost 150 baht. Find the cost of a pencil and a pen.

Don't try to guess … work it out.

If P is the price of a pen and p is the price of a pencil …

$$P = p + 30 \qquad\qquad \text{…. [1]}$$

$$P + 2p = 150 \qquad\qquad \text{…. [2]}$$

Putting the value of P from equation 1 in equation 2 gives …

$$(p + 30) + 2p = 150$$

$$3p = 120$$

and … $\qquad p = 40$ Baht

now … $\qquad P = 40 + 30$

$$= 70 \text{ baht}$$

There are other ways to do this calculation: see below.

Second method

If P is the price of a pen and p is the price of a pencil …

$$P = p + 30 \qquad \text{…. [1]}$$

$$P + 2p = 150 \qquad \text{…. [2]}$$

We have two equations in two unknowns: P and p. We want values of P and p to make both equations true at the same time. A method that works for all equations of this type, is to multiply one equation by something that leaves only one of the unknowns when the two equations are added.

Multiplying equation 1 by 2 gives …

$$2P = 2p + 60 \qquad \text{…. [3]}$$

Adding equations 2 and 3 gives …

$$3P + 2p = 2p + 210$$

$$so \; … \;\; P = 70 \text{ Baht}$$

Rearranging equation 1 … $\quad p = 70 \text{ - } 30$

$$= 40 \text{ Baht}$$

This method is taught in High Schools.

Equations 1 and 2 are called *linear* equations.

1 Linear equations don't have powers or products of the unknowns.

2 *Linear* equations in two variables can be plotted on the Cartesian plane as straight lines. Plotting lines and finding their intersection is another way to find the values of P and p.

Graphical method

Normally the variables on the plane are y and x, but in the answers above we chose to use p and P, so let's solve this on the plane using p and P.

If P is the price of a pen and p is the price of a pencil …

$$P = p + 30 \qquad\qquad \text{.... [1]}$$

$$P + 2p = 150 \qquad\qquad \text{.... [2]}$$

We can plot these straight lines if we know the intercept on the vertical axis and the slope (rise/run).

Rewriting equations 1 and 2 with p on the left of the equals gives …

$$p = P - 30 \qquad\qquad \text{.... [3]}$$

$$p = -{}^1\!/_2\, P + 75 \qquad\qquad \text{.... [4]}$$

The Cartesian axes are labelled with p and P replacing y and x.

Values of p are plotted on the vertical axis.

Values of P are plotted on the horizontal axis.

The slope of line 3 is plus one and
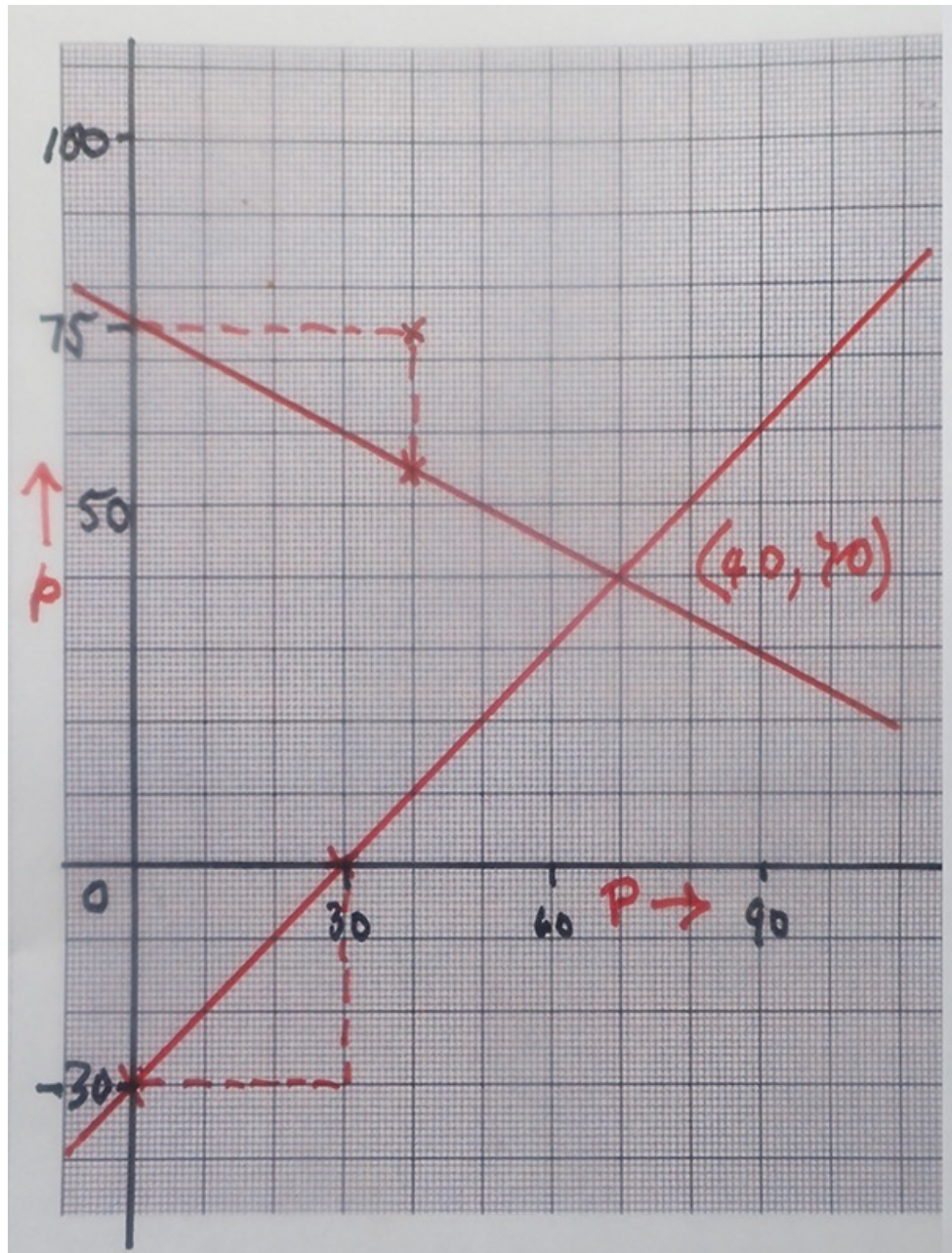the intercept on the vertical p axis is -30.

The slope of line 4 is minus one half and
the intercept on the vertical p axis is +75.

The lines are drawn on the graph below.

Two lines are drawn in red on the plane with Cartesian coordinates (p, P).

$$p = P - 30$$

$$p = -^1/_2\, P + 75$$

The intersection of the lines (*where both equations are true at the same time*) is the point $(40, 70)$, confirming the answers above.

The pencil costs 40 Baht and the pen costs 70 Baht.