Descartes

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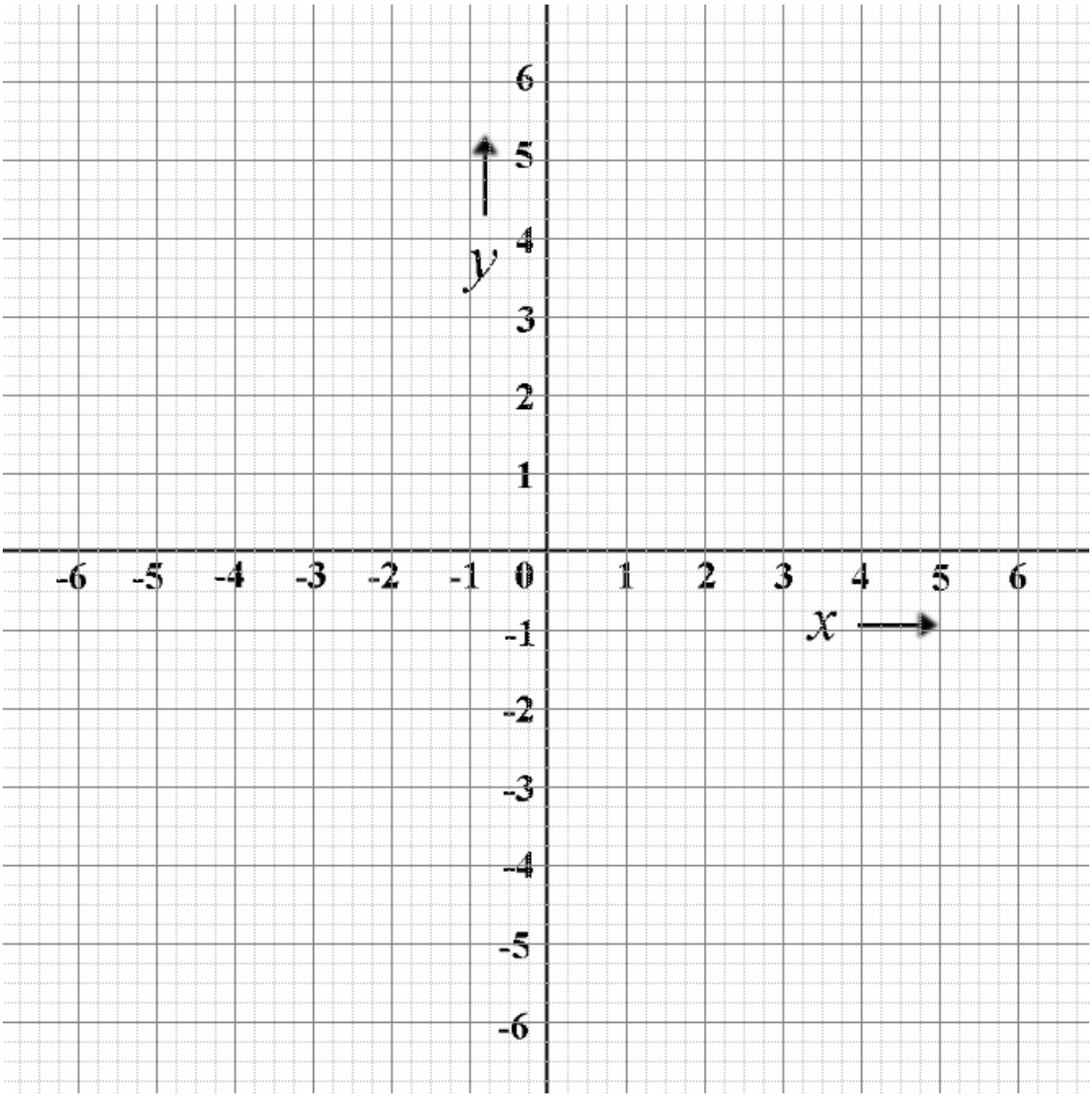
Mathematics in the time of the ancient Greeks like *Pythagoras* and *Erosthenese of Alexandria* and for early Europeans like *Issac Newton* was a matter of geometry not algebra. Newton's relationships and proofs are in modern text books as algebraic expressions, but his original proofs were geometric. The equivalence of algebra and geometry was first shown by Descartes.

> *René Descartes was a clever man who died in 1650. He had money, and the time to think and write and come up with mathematical ideas. European scholars wrote in those times in Latin, but Descartes said he wrote in French so that all who had good sense, including women, could read his work and learn to think for themselves. He was well ahead of his time.*

René Descartes

> *His name in Latin was Renatus Cartsesius and the graph he invented to cover a flat surface to connect Geometry with Algebra is called the* ***Cartesian Plane****.*

The Cartesian Plane



Two perpendicular number lines are placed on the plane.

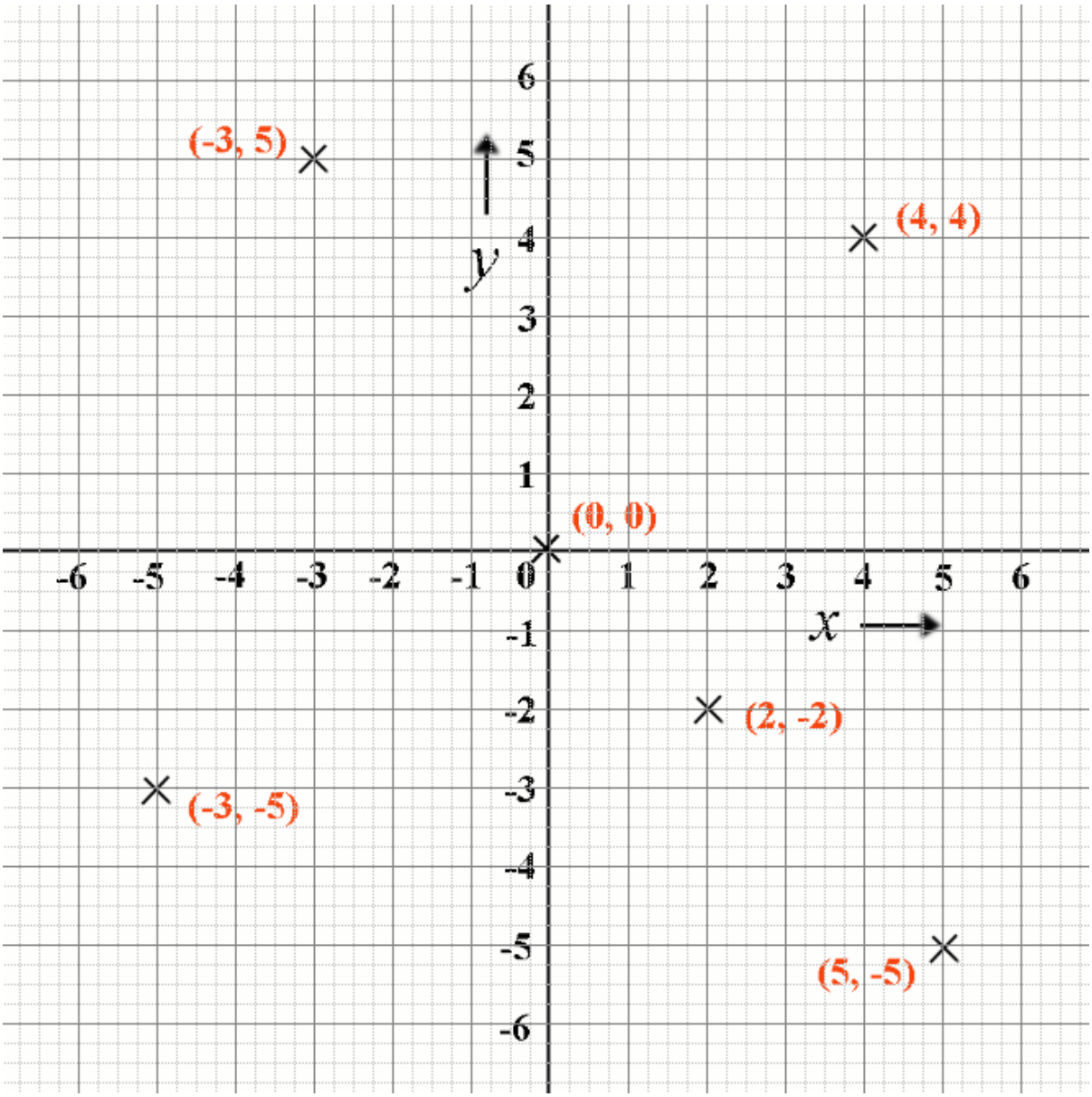
The number lines are called axes.

The horizontal axis is labelled x and the vertical axis is labelled y.

A point on the plane is defined with two numbers (*x, y*) *called an ordered pair.*

The axes cross at the origin: the point (0, 0).

Ordered pairs and Quadrants



Six points on the plane are marked with crosses and labelled in red with ordered pairs.

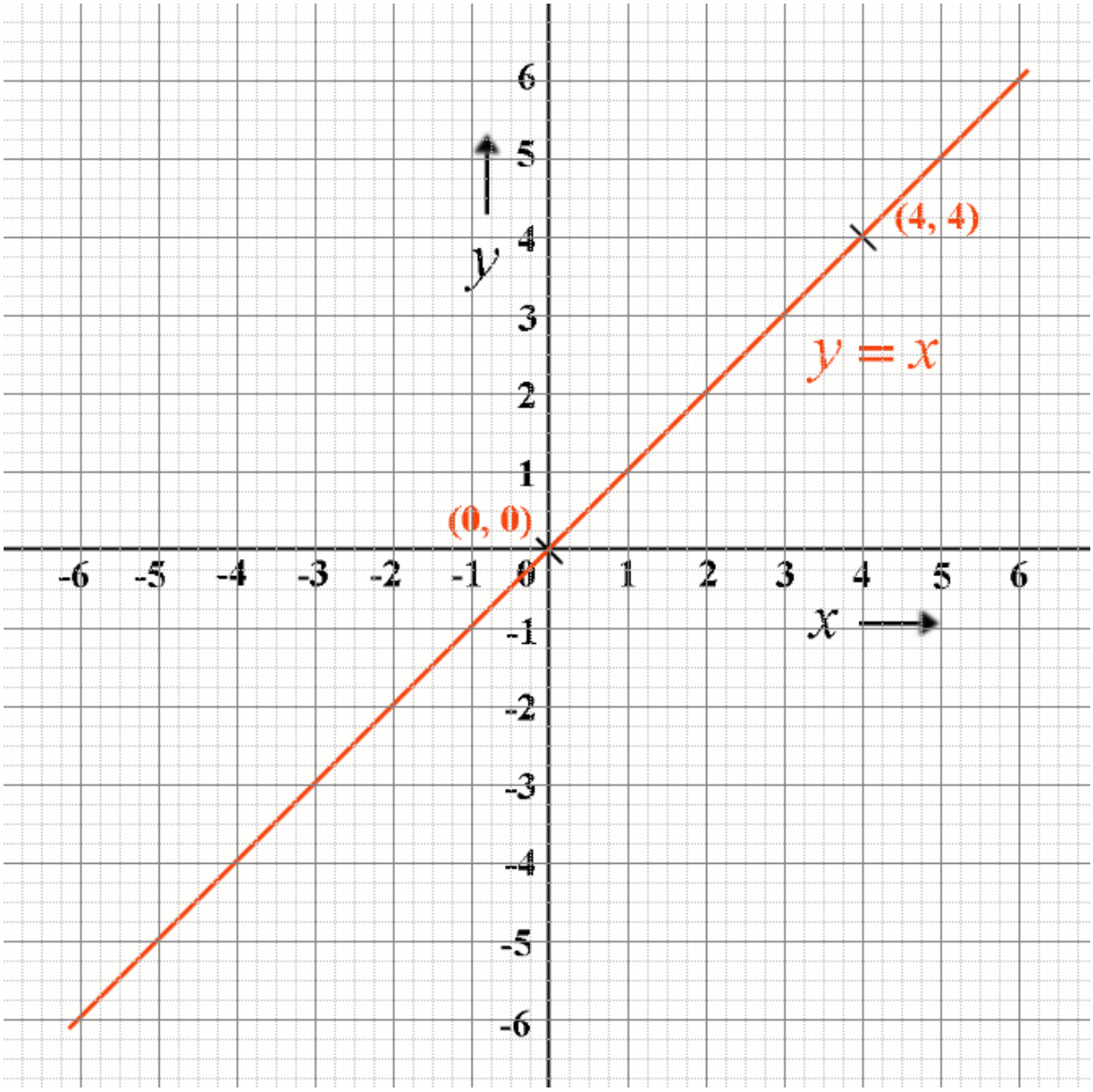
The point (4, 4) *is in the first quadrant.*

The point (-3, 5) *is in the second quadrant.*

The point (-3, -5) *is in the third quadrant.*

The points (2, -2) *and* (5, -5) *are in the fourth quadrant.*

The line $y = x$



The line $y = x$ passes through the origin, (0, 0)*, and all points for which $y = x$.*

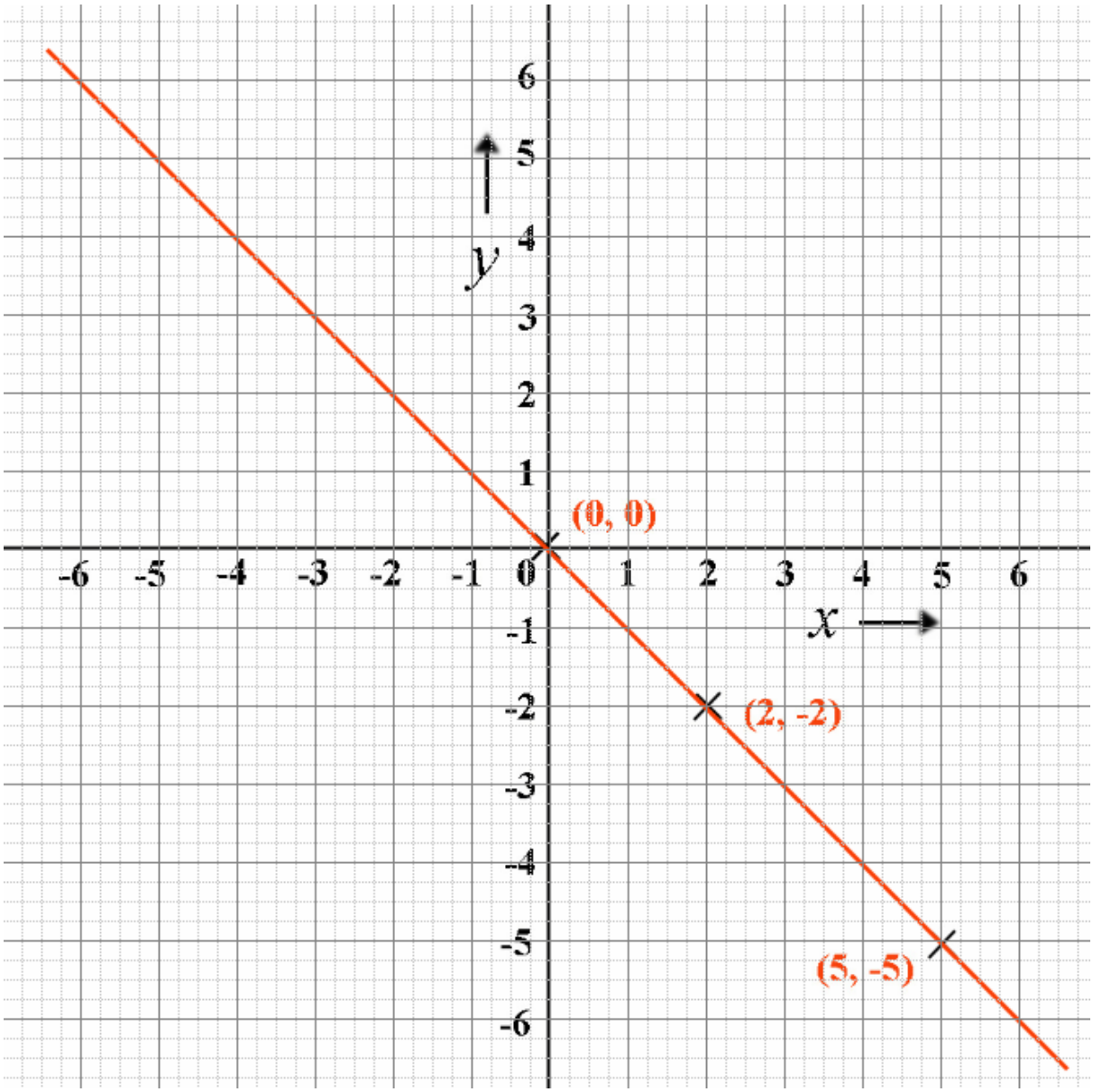
The point (4, 4) *lies on the line.*

Look along the line and name other points through which it passes.

The line slopes upwards to the right at an angle of 45°.

The ***gradient*** *of the line is said to be* +1.

The line $y = -x$



The line $y = -x$ passes through the origin, (0, 0)*, and all points for which $y = -x$.*

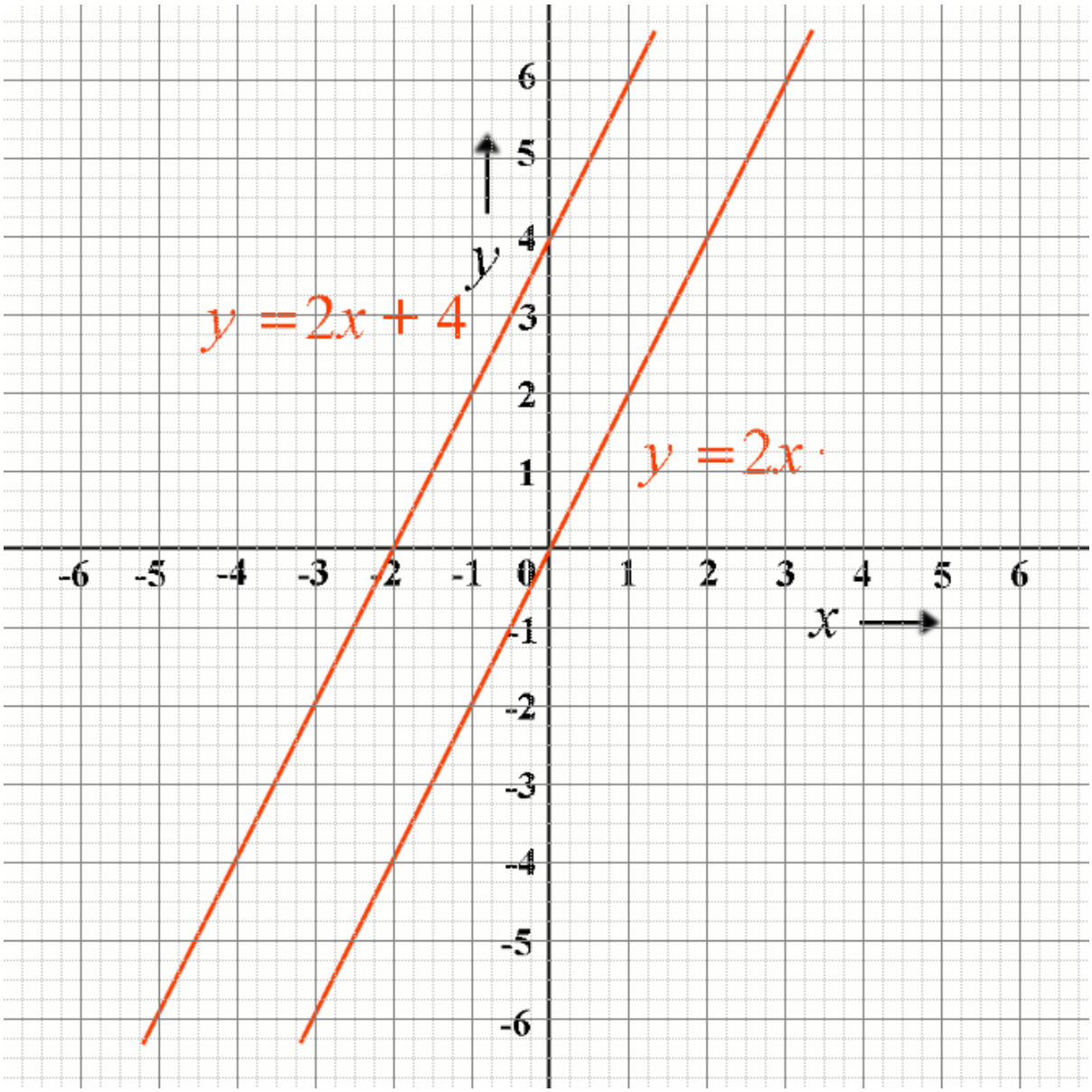
The points (0, 0), (2, -2) *and* (5, -5) *are on the line.*

Look along the line and name other points through which it passes.

The line slopes downwards to the right at an angle of 45°.

The ***gradient*** *of the line is said to be* -1.

A line that does not pass through the origin



y = 2x ... passes through (0, 0) *and has a gradient of* +2
y = 2x + 4 *... passes through* (0, 4) *and has a gradient of* +2

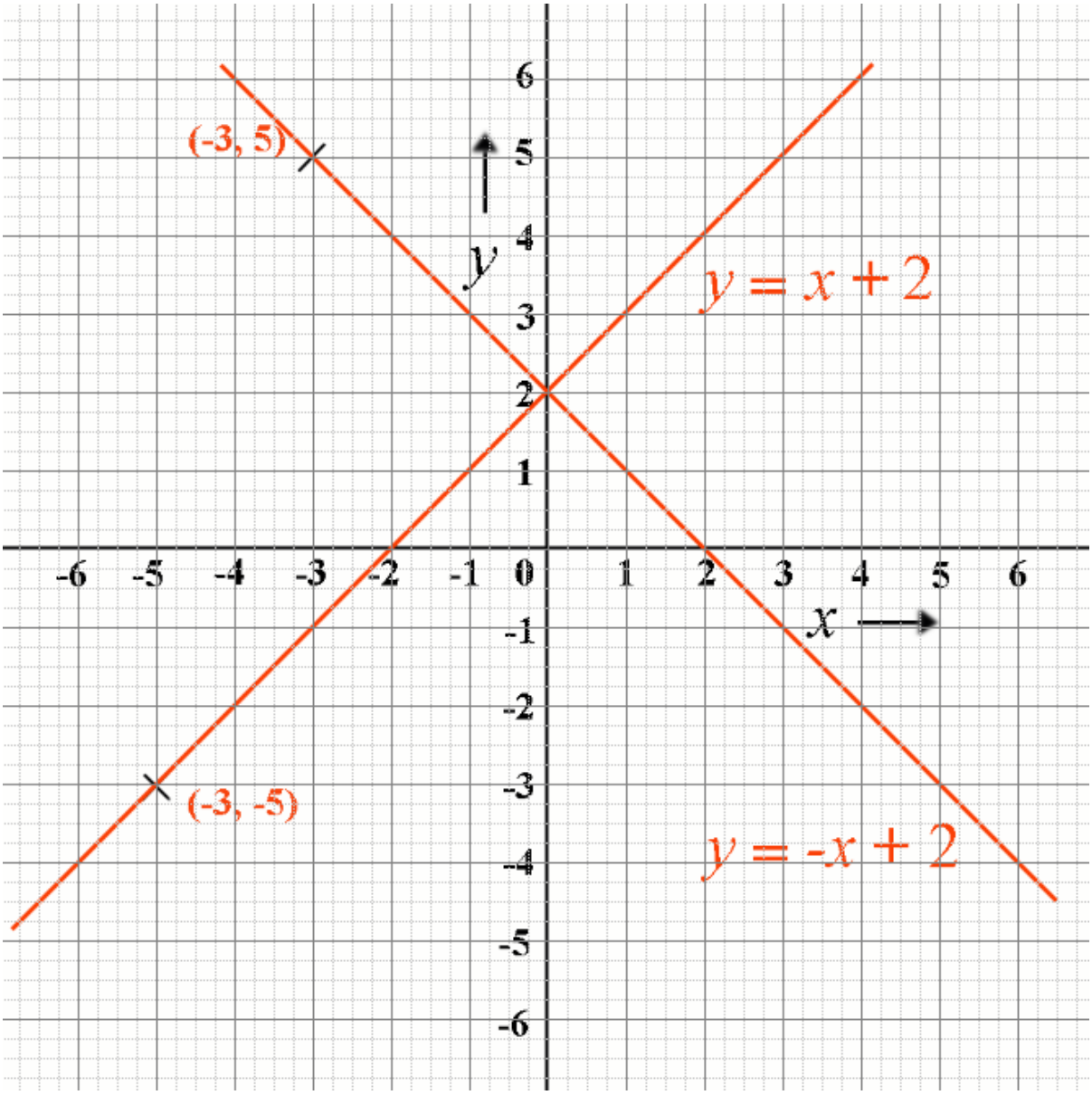
The equation of a straight line can always be written as …

$$y = mx + c$$

… where *m* and *c* are constants.

The value of m is the ***gradient*** *of the line and c is the* ***intercept*** *(the crossover point) on the y axis.*

More lines that do not pass through the origin



The line $y = x + 2$ *passes through the point,* (0, 2), *and all other points for which* $y = x + 2$.

What is the gradient and what is the y intercept of this line?

The line $y = -x + 2$ *passes through the point,* (0, 2), *and all other points for which* $y = -x + 2$.

What is the gradient and what is the y intercept of this line?

Linear equations in standard form … $y = mx + c$

Look carefully at the five equations below. They are **linear** equations: they contain only two variables, x and y, and there are no products of the variables, no x^2, y^2 or xy terms.

$$x = y$$

$$y + x = 0$$

$$2y - 3x + 4 = 0$$

$$y/2 + x/2 = 4$$

$$2(x + 2) = 2(y + 2)$$

Each equation can be rewritten with y on the left hand side. If we do that, we have values of m and c. We can plot the lines on the Cartesian plane using just those values.

In standard form … $y = mx + c$ … the equations are …

$$y = x$$

$$y = -x$$

$$y = 3/2\, x + 2$$

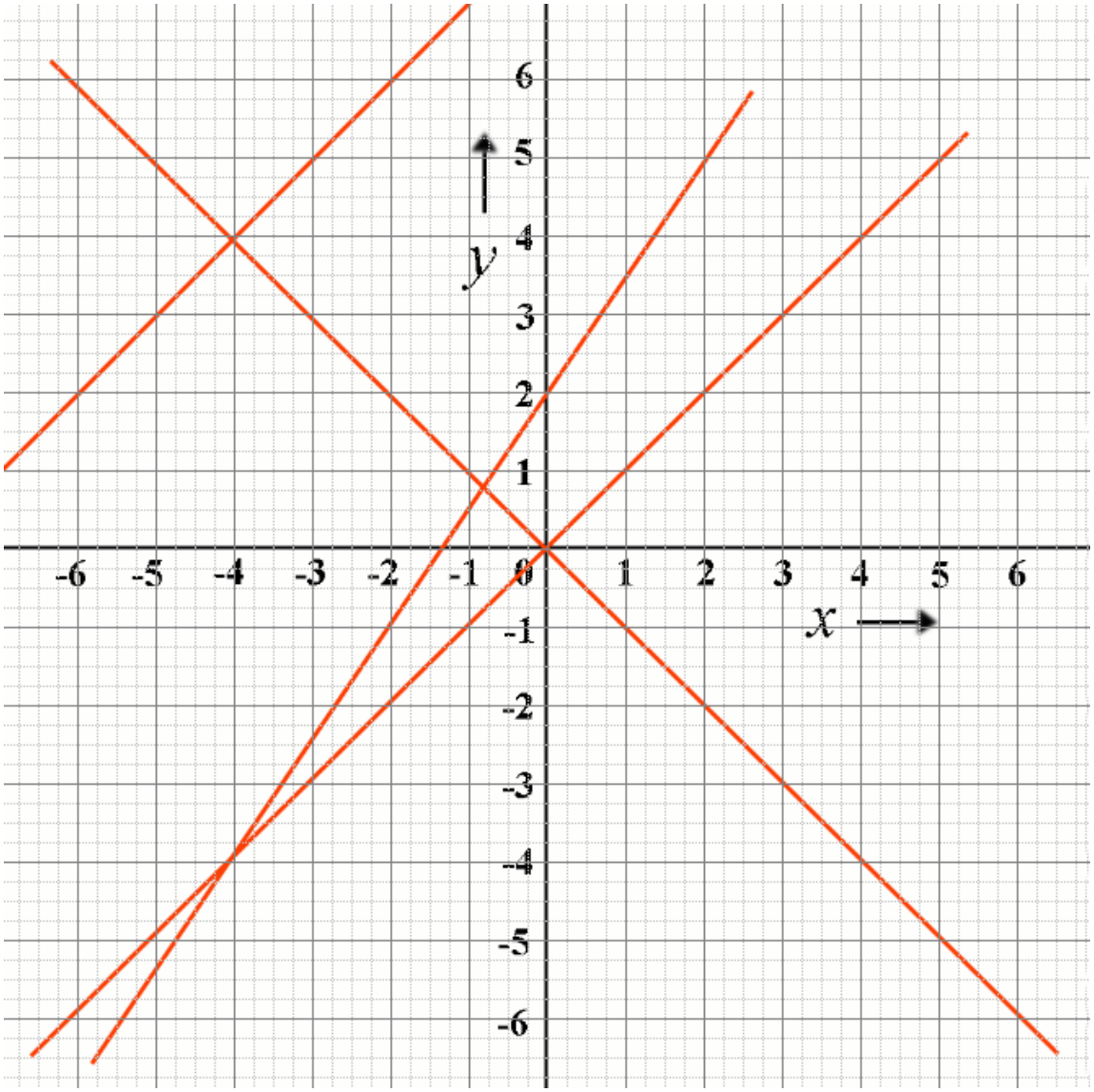
$$y = x + 8$$

$$y = x$$

The first and last equations are identical.

The four lines can now be drawn on the plane using the values of c (the y intercept) and m (the gradient) found by inspection.

The four lines on the Cartesian plane



In standard form … $y = mx + c$ … the four lines are …

$$y = x$$

$$y = -x$$

$$y = {}^{3}/_{2}\, x + 2$$

$$y = x + 8$$

The reader is asked to match the lines with their equations.