

Pythagoras

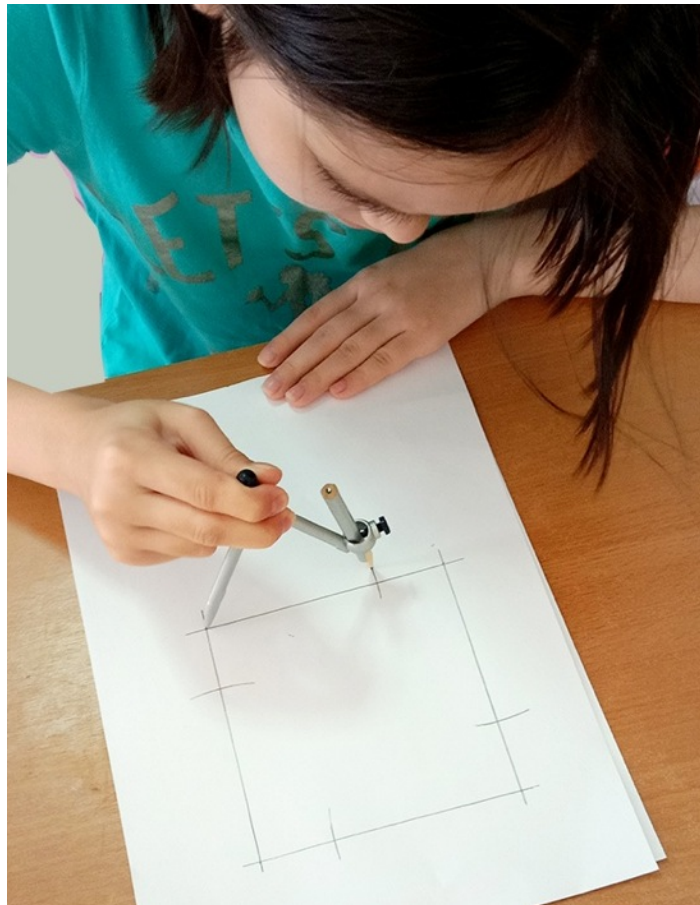
Shannon and Ian Jacobs

Pythagoras of Samos was an ancient Greek philosopher and mathematician who lived around 500 BC. He is most famous today for the theorem about the lengths of the sides of a right angled triangle that bears his name.

In an right angled triangle, where a is the longest side, the squares on the sides of the triangle, a , b and c , have areas that satisfy the equation ...

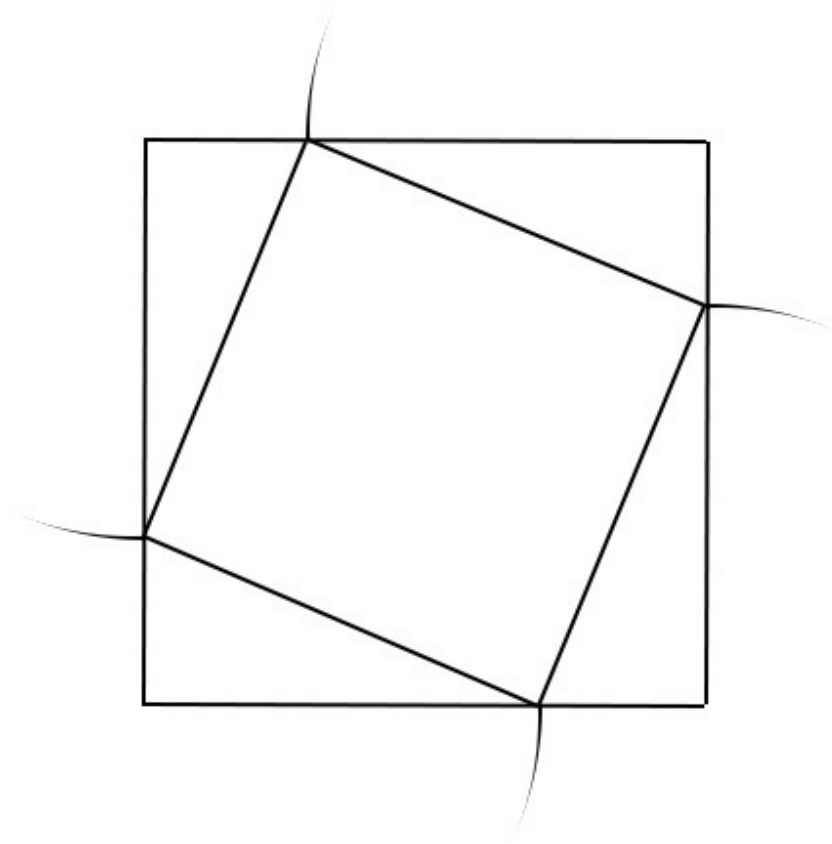
$$a^2 = b^2 + c^2$$

For a simple proof, draw a square.

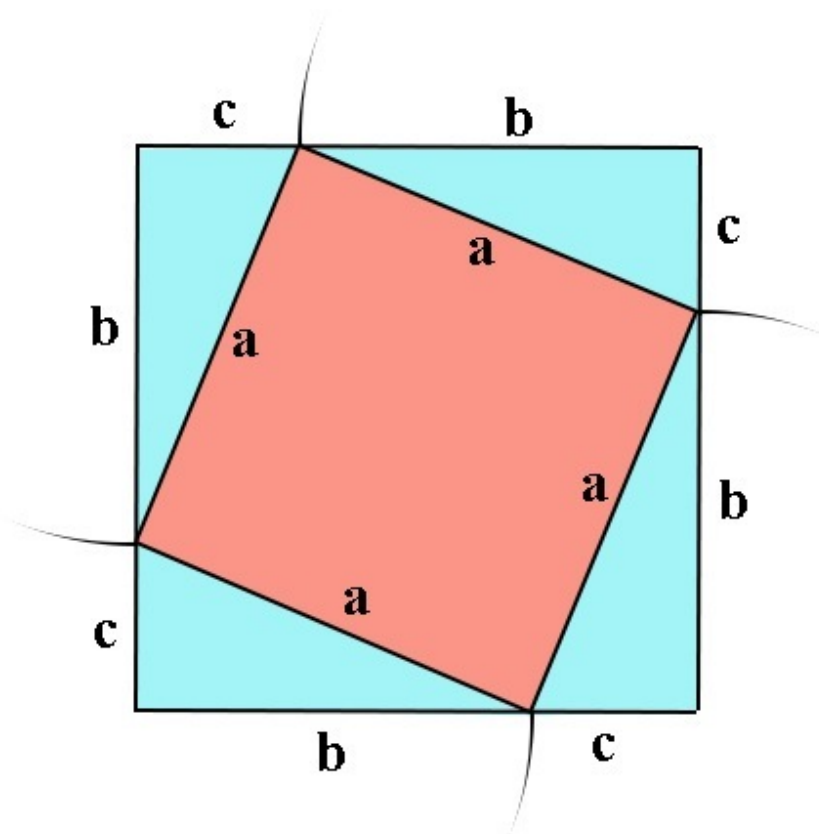


Put the point of a compass on a vertex, and draw an arc somewhere along a side. Go around the square and repeat that for each side.

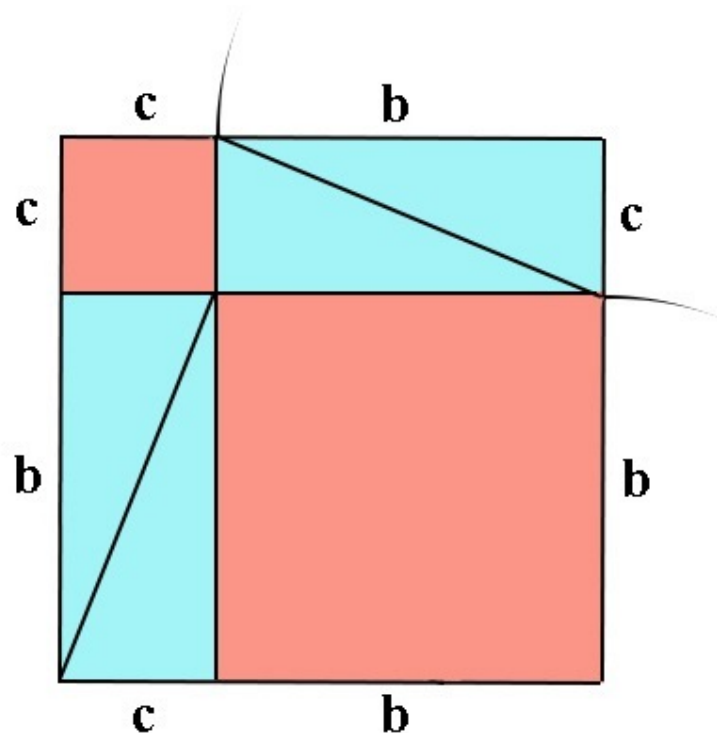
Join the points where the arcs cross the sides to form four identical (congruent) triangles inside the square.



The sides of the triangles have been labelled a , b and c and the shapes have been colour coded. Explain why the central pink area is also a square.



The blue triangles have been rearranged.



Inspecting both coloured diagrams shows at once that the square on side a has the area of the sum of the squares on sides b and c .

$$a^2 = b^2 + c^2$$

This result can also be found by writing down the area of the large square in two different ways.

The area A is given by ... $A = a^2 + 2bc$

... and also by ... $A = (b + c)(b + c)$

Equating these expressions and multiplying out the brackets gives ...

$$a^2 + 2bc = b^2 + 2bc + c^2$$

Subtracting $2bc$ from both sides gives ...

$$a^2 = b^2 + c^2 \quad \dots \text{ as above.}$$