Numbers [2]

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> *"Hi Shannon.* Dad here *. Remember: I wanted to show you another way to add the numbers from 1 to 100? This is not mine, it comes from a story that's been told and retold in mathematics lessons for a long time. One version of the story goes like this …"*

"In a school in Germany many years ago there was a boy of 8 by the name of Carl Friedrich Gauss. He was apparently a talkative child and a trial to his teacher who was teaching slower kids and trying to keep Carl quiet. The teacher gave a long problem: add the numbers from 1 to 100. It is not clear whether he gave the problem just to him or to everyone in the class but that doesn't matter so much. What matters is that young Gauss came back with the answer (5050) in his head without working on paper. The story-tellers all agree on how he did that. He added the numbers twice."

$$1 + 2 + 3 + \ldots\ldots + 98 + 99 + 100$$
$$100 + 99 + 98 + \ldots\ldots + 3 + 2 + 1$$

He added the two lines ….

$$101 + 101 + 101 + \ldots\ldots + 101 + 101 + 101$$

The answer to that is 100x101 which is 10100.

Half of that is the answer he wanted .… 5050.

Since the number of numbers (n) is 100, the formula for that is …

$^1/_2 n(n+1)$ … which we know from my method.

The truth is, a child who could become a mathematician in adult life (Gauss became a wonderful mathematician in his time) will probably be quicker at mathematics than his teacher. (*Dad has his own story about that, which I will tell you later*).

Let's use Gauss's method to find the sums of odd and of even numbers.

Odd numbers

$$1 + 3 + 5 + 7 + 9$$
$$9 + 7 + 5 + 3 + 1$$

Adding the two lines gives 10 + 10 + 10 + 10 + 10

That is 5x10, and half of that is 5x5, the answer we want.

Since n is 5 that is n^2, which we know from my method.

Even numbers

$$2 + 4 + 6 + 8 + 10$$
$$10 + 8 + 6 + 4 + 2$$

Adding the two lines gives 12 + 12 + 12 + 12 + 12

That is 5x12, and half of that is 5x6, the answer we want.

Since n is 5 that is $n(n+1)$, which we know from my method.

Gauss's method and my method give answers and relationships in the same number of lines. You decide which method you prefer. I prefer mine (of course).

The next thing Dad said he would show me was how to how to *prove* that the formula for the sum of n whole numbers beginning with 1 is $^1/_2 n(n+1)$ for any value of n. Every time I do an addition that is true, so I'm happy with that, but I remember that he said that a mathematician would do it another way.

He did it carefully. I followed it: I understood it: so here is what he did.

The sum of the numbers from 1 to any value of *n*

Suppose we know that for some value of *n,* which we will call *k,* that the sum of the numbers from 1 to *k* is $^1/_2 k(k+1)$.

What is the sum if *k* is increased to $(k + 1)$?

The sum must be what it was, plus the next number in the sequence.

The new sum is $^1/_2 k(k+1) + (k+1)$

$$^1/_2 k(k+1) + (k+1) = {}^1/_2[k(k+1) + 2(k+1)]$$

$$= {}^1/_2[k^2 + k + 2k + 2]$$

$$= {}^1/_2[k^2 + 3k + 2]$$

$$= {}^1/_2(k + 1)(k + 2)$$

… which is $^1/_2 k(k+1)$ with k replaced by $(k + 1)$

The tricky bit here is multiplying out the brackets and putting brackets in again. I *can* do that, but I have to think about it carefully, so I've put in each line and not gone straight to the answer like in a text book.

So: if the relationship is true for one value of *n* it is true for all values of *n*.

This is called a proof by *induction*.

For practice I just did the proof of n^2, for the sum of the of the odd numbers, and $n(n+1)$, for the sum of the even numbers.

You could do those for yourself.