Numbers

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I'm camping with families from my first school and I have this busy-work. Pages of homework sums to do by next week.

I asked Dad about whether I really have to do them.

> "*You told me 'Calculators' were people who sat all day at desks in big rooms being paid to add and subtract. Nobody gets paid to do that, so do we really need do this? My phone has a calculator.*"

This is what I remember of what he said.

> *"Sorry you're stuck in arithmetic but that's not anyone's fault. Schools teach mathematics to try to train you to think logically. It doesn't work that well. Adults everywhere have strange hopes and beliefs but Science is gradually creeping into Religion and Politics."*
>
> *"If you don't learn a few tables now and practice sums you'll miss out on things later that are more interesting. Don't let this ruin you for learning. Next week we'll put on the "air" upstairs and do clever adding and you'll feel better."*

So: I calmed down and went on with my silly life on Instagram.

Next week he gave me five numbers to add … “*without working*”.

The odd numbers … 1 + 3 + 5 + 7 + 9

I did it this way

$$1 + 3 + \textcircled{5} + 7 + 9$$

Associate two at a time and count

10 + 10 + 5 = 25

The sum is 5^2.

I do it this way because we play a game in the car. We have races to add the four digits on passing number plates. I do it by association.

That’s our number plate … the sum is 10 + 10 = 20 … by association.

Odd numbers

Let's try 10 odd numbers

$$1 + 3 + \textcircled{5} + 7 + 9 + 11 + 13 + \textcircled{15} + 17 + 19$$

Associate and count

$$10 + 10 + 30 + 30 + (5 + 15) = 10x10$$

When n (the number of odd numbers) is 10, the sum is n^2.

Lets check when $n = 7$

$$1 + 3 + 5 + \textcircled{7} + 9 + 11 + 13 = 14 + 14 + 14 + 7 = 7x7$$

I asked Dad if this *proves* that the sum of the first n odd numbers is always n^2. I remember this from what he said.

> "It depends who you ask. Physics types use rules they find to be true. The laws of physics are like that. They accept a rule as always true until they find a case where it's not. They do the same with mathematics. If a rule works they use it. A mathematician might say that to *prove* this we need to show that if the rule is true for some value of n, then it's always true if n is increased to $(n + 1)$."

I want to do other things first so I asked him to wait on that one.

Even numbers

$$2 + 4 + \textcircled{6} + 8 + 10 = 12 + 12 + 6 = 5x6$$

When $n = 5$ the sum of the even numbers is $n(n + 1)$. That makes sense. Adding 1 to each odd number gives even numbers and $n(n + 1) = n^2 + n$.

Natural numbers

Let’s do this first with $n = 10$

$$1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10$$

$$= 5 \text{x} 11$$

$$= {}^{1}/_{2}\,(10 \text{x} 11)$$

The sum is 55. That is ${}^{1}/_{2}\, n(n + 1)$.

Let’s do this with $n = 15$

$$1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11 + 12 + 13 + 14 + 15$$

$$= (7 \text{x} 16) + 8$$

$$= {}^{1}/_{2}\,(15 \text{x} 16)$$

The sum is 180: again ${}^{1}/_{2}\, n(n + 1)$.

Dad says he’d like to show me other ways to do this, but not right now.

Let’s do this my way with $n = 100$

$$1 + 2 + \cdots\cdots + 50 + 51 + \cdots\cdots + 99 + 100$$

$$= 50 \text{x} 101$$

$$= {}^{1}/_{2}\,(100 \text{x} 101)$$

The sum is 5050: again ${}^{1}/_{2}\, n(n + 1)$.