Adding fractions with the big gun

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My little sister (5) can add any two single digit numbers. Dad says he thinks he could always do that. He can't remember learning to do it.

When we add (7+2) we're really just counting-on along the number line. Count to 7 from zero, and count *on* 2 more, which brings us to the 9. When we subtract 2 from 7 we count to 7 and then *back* 2, which brings us to 5.

We learn strategies to add big numbers and forget that what we're doing is counting-on but helps to remember that when we come to adding fractions.

$$^1/_2 + ^1/_3 = ?$$

To put the half and the third on a number line so we can count-on we can write the half as three sixths and the third as two sixths …

$$^3/_6 + ^2/_6 = ^5/_6$$

Just count-on. Three sixths and two more sixths make five sixths. In math books the six is called a *common denominator*: big words for something simple. The six is just two times three. A common denominator is easy to find. All you have to do then is figure out the *numerators* (the numbers on the top lines of the new fractions). You have to practice that in school: I don't want to do that here.

I asked Dad to show me how to write down the answer for any sum of fractions: for something like this shocker.

$$^1/_3 + ^1/_5 + ^1/_7 - ^1/_{11} + ^1/_{17}$$

I wonder if my teacher could do that?

I could challenge him to a race!

Dad wrote a line of three fractions like this with letters replacing the numbers and he put a minus sign in there.

$$a/A + b/B - c/C$$

We could make each fraction into a whole number by multiplying everything by *ABC*. We will do that because we know how to add whole numbers.

$$ABC\left[a/A + b/B - c/C\right] = aBC + AbC - ABc$$

The sum on the right will be *ABC* times bigger than we want. To get the right answer we divide the sum on the right by *ABC*.

$$a/A + b/B - c/C = \frac{[aBC + AbC - ABc]}{ABC}$$

The fraction on the right is the answer we want!

Example:

Simple examples are best. We pick one to which we know the answer by inspection.

Find the sum of the three fractions … $1/2 + 1/4 - 1/8$

Substitute the numerators and denominators in the formula carefully.

$$1/2 + 1/4 - 1/8 = \frac{(4x8 + 2x8 - 2x4)}{2x4x8}$$

To simplify that answer cancel 8 and add the terms in the bracket.

$$= 5/8$$

To add five fractions in one step extend the formula to ten letters.
Note that one letter in each term in the numerator is a lower case letter.

$$a/A + b/B + c/C + d/D + e/E = \frac{[aBCDE + AbCDE + ABcDE + ABCdE + ABCDe]}{ABCDE}$$

Let's do the shocker … $1/3 + 1/5 + 1/7 - 1/11 + 1/17$

Substituting numerators and denominators in the formula with a dot to represent multiply signs to save space gives …

$$\frac{(5.7.11.17 + 3.7.11.17 + 3.5.11.17 - 3.5.7.17 + 3.5.7.11)}{3.5.7.11.17}$$

That is the answer, but have you got a calculator on your phone?

I make it 12647/19635. *You could check that*. The original denominators were prime numbers. The common denominator has no other factors. No further simplification is possible. I did say it was a shocker.