Random packing

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Dad asked me to bring a bottle of sand back from Ko Lipe in the South of Thailand.

I put a half-litre bottle of white sand in my suitcase in the plane going home.

When I got home I found the mass of my bottle of sand was 800 g.

Density

Density is the mass of an object divided by it’s volume. The density of our sand is the mass (800 g) over the volume (500 cc).

The density of our sand is 800/500 = 1.60 g/cc.

Note: that value (1.60) is not exact. The empty bottle had a mass of 20 g, the balance might not have been perfectly accurate, and we don't know how full the bottle needed to be to have 500 cc in it. The sand is probably made of quartzite. The density of quartzite (from the web) is 2.6-2.8 g/cc. Other possible rocks have higher densities. Why is the density of my sand only 1.6 g/cc? Was there something wrong with the balance?

1 The measured mass of the half litre bottle of water was close to 500 grams. The balance was reading correctly. The density of water is 1g/cc.

2 I suggested we try putting stones in the bottle instead of sand. Dad gave me a funny look and said, "*Why didn't I think of that*?"

3 An 800 gram pile of smooth irregular stones fitted in the bottle: just.

4 To find the empty space I added enough water to fill a stone-filled bottle and a sand-filled bottle and massed them.

The mass of each bottle topped up with water was 1000 g = (800+200) g. The same amount of water (200 cc) was added to each bottle. The total empty space between the stones and the sand was the same in each case (200 cc) and the volume of the rock (quartzite) was 500-200 = 300 cc.

The density of the rock = 800/300 = 8/3 = 2.67 g/cc

There is no mystery here. The density of quartzite is 2.6-2.8 g/cc. The stones are made of quartzite or a rock of very similar density.

The 40% rule

It is found that close to 40% empty space is left when spheres of the same size are poured into a container. If the container is vibrated, the shaking reduces that to about 36%. The members are not exact to better than 1%. For a table try … *en.wikipedia.org › wiki › Random_close_pack*.

> *Imagine drawing a full page diagram of same-size circles touching randomly and then shrinking the whole page. The **ratio** of areas, the area inside the circles to the area of the page, **stays the same**. Many people get this wrong because they don't think carefully.*

Our stones were smooth, only approximately the same size, and not round like marbles.

Our sand grains (above) were not round and there were a few larger ones. There were some tiny ones that fitted in spaces.

The 40% empty-space rule, that's true for marbles of the same size, applied almost equally well to our stones and our sand. The irregular shape and size of our stones and sand grains made very little difference.