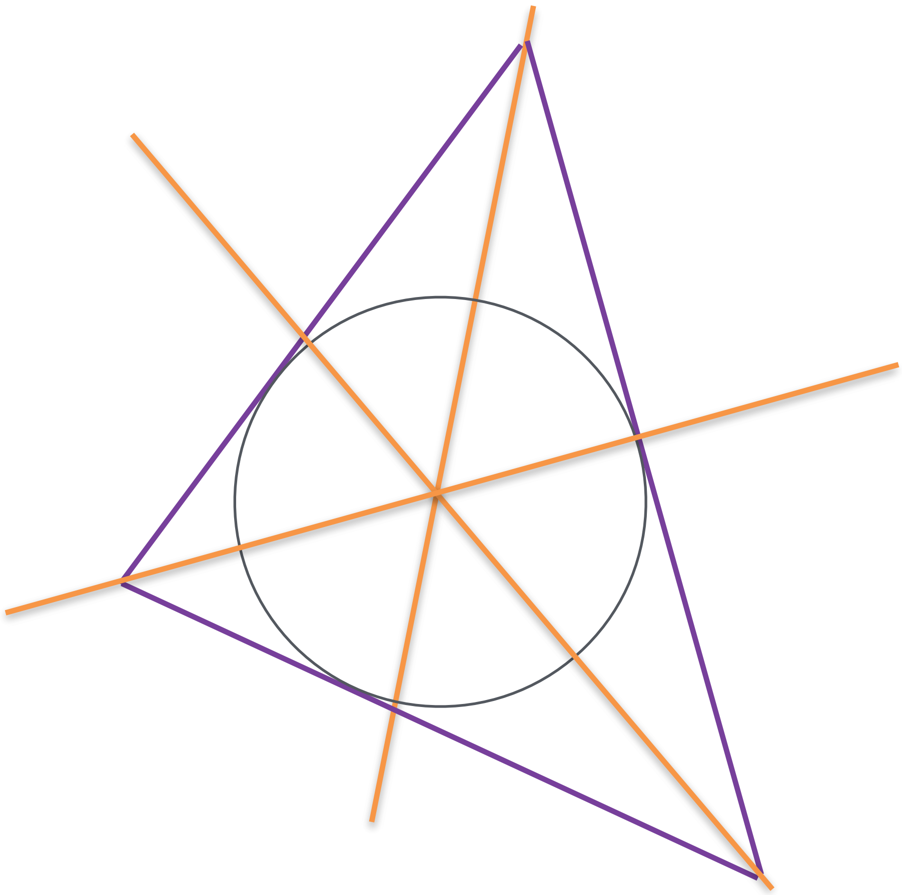
The incircle

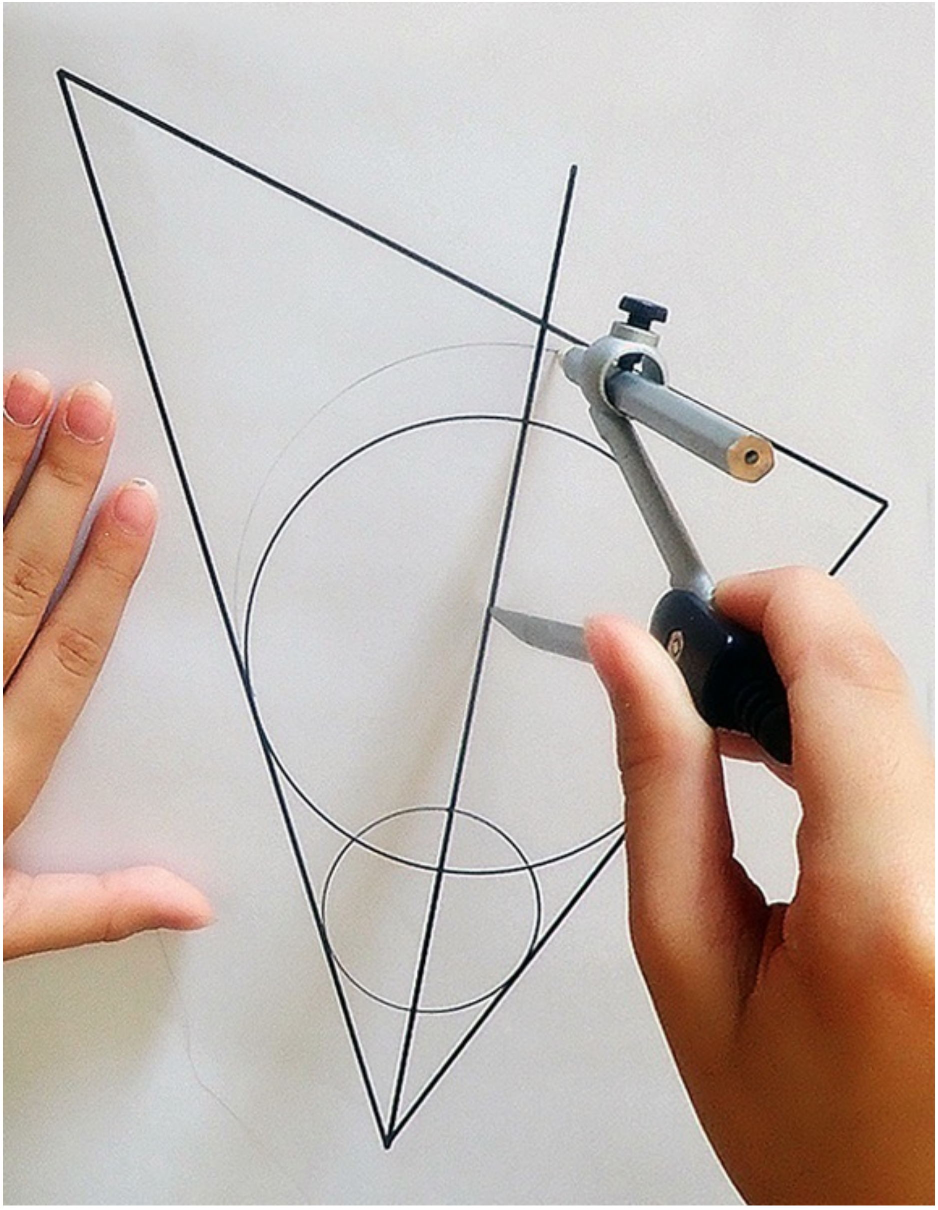
Ian and Shannon Jacobs



The three angle bisectors intersect at the centre of the *incircle* that touches all three sides of the triangle.

*To **prove** this (verify this construction), imagine a circle drawn near a corner of a triangle (vertex) with the centre on an angle bisector that just touches the two adjacent sides. This can always be done because the radius of the circle is the perpendicular distance from a side to the centre of the circle on the angle bisector. Now: make that circle bigger and move the centre away from the vertex along the bisector until it touches all three sides and becomes the incircle. The incircle can be formed in this way on any one of the three angle bisectors. There is only one incircle, so the angle bisectors must meet at a point, which is the centre of the incircle.*

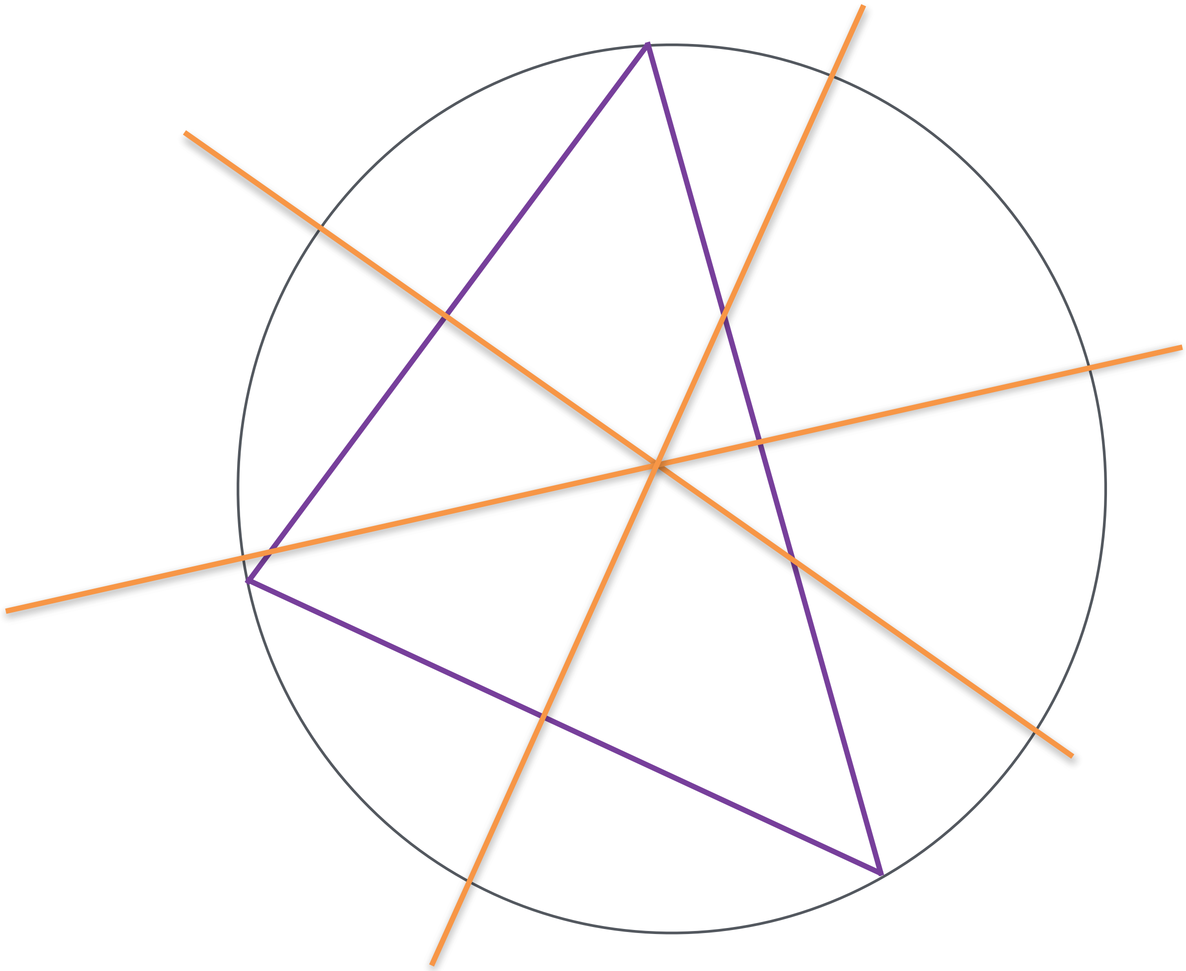
The reasoned argument above is sufficient, but if you're not sure, you can do what I did. Draw a triangle on paper with a ruler and carefully construct an angle bisector with a compass. For this to be convincing your angle bisector must be accurate.



Draw circles with centres on the angle bisector that touch two sides of the triangle. Move the centre along the bisector until you have the *incircle* that just touches all three sides of the triangle. The centre of my largest circle is not quite far enough from the lower vertex.

When you have it right, draw the other two angle bisectors that will cross at the centre of your incircle.

The circumcircle

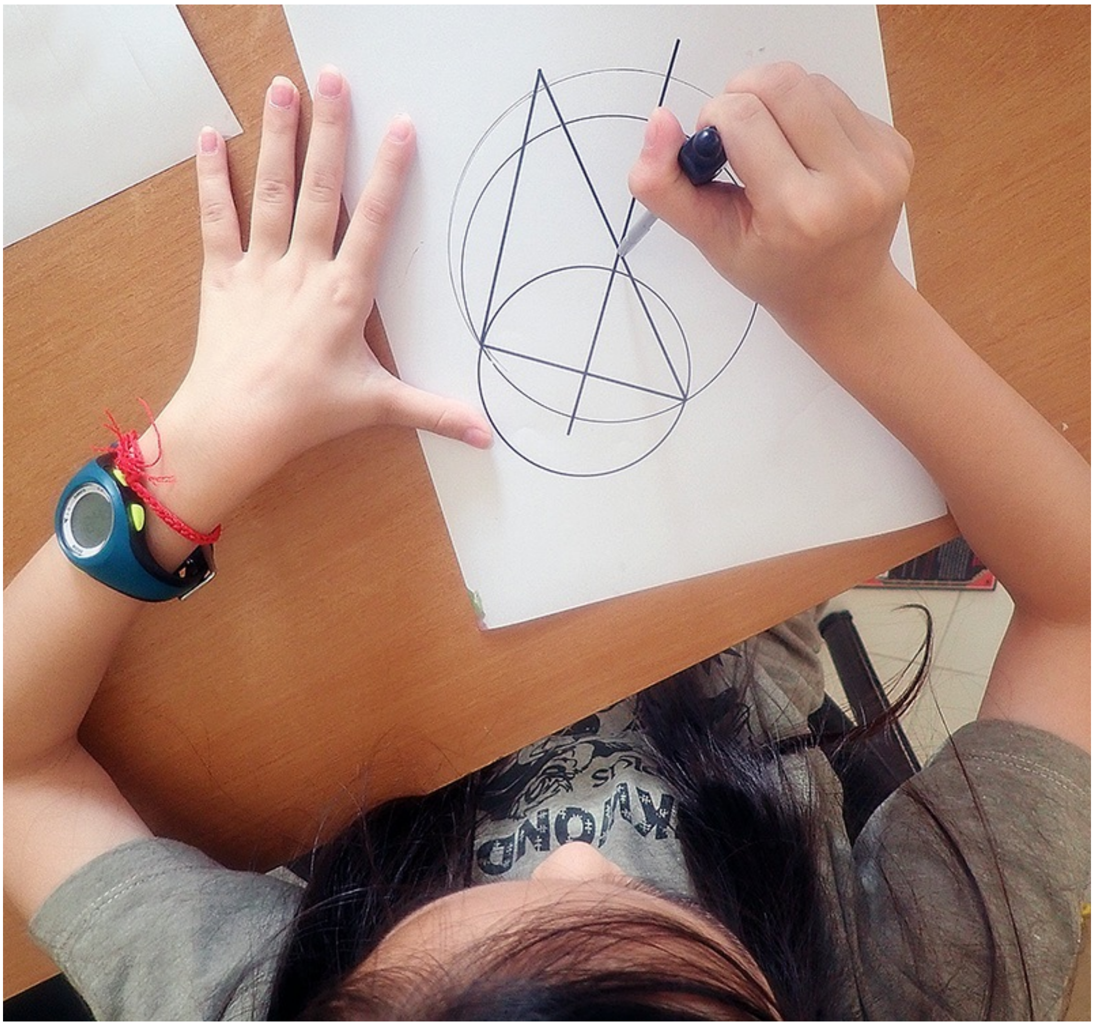


Perpendicular bisectors of the sides of a triangle intersect at the centre of the *circumcircle,* passing through all three vertices (points). Find the equal line segments and the right angles on the diagram.

To ***prove*** *this (verify this construction), imagine a circle centred on the intersection of a side and its perpendicular bisector, with a radius of half the length of the side. Now make the circle bigger and move the centre along the bisector until it becomes the circumcircle that passes through all three vertices. Because the circumcircle can be formed in this way on any bisector, and there is only one incircle, the perpendicular bisectors must meet at a point, which is the centre of the circumcircle.*

The reasoned argument above is sufficient, but if you're not sure, you can do what I did. Draw a triangle on paper with a ruler and carefully construct the perpendicular bisector of a side with a compass. For this to be convincing your perpendicular bisector must be accurate.

Look at the image: find the equal line segments and the right angles.



Draw circles with centres on the perpendicular bisector that pass throughout the two lower vertices of the triangle. Move the centre of your circles along the bisector until you have the *circumcircle* that passes through all three vertices of the triangle. The centre of my largest circle is not quite far enough away from me. Try again.

When you have it right, draw the other two perpendicular bisectors that will cross at the centre of your circumcircle.